Fast, Compact, and Constant-time Discrete Gaussian Sampling over Integers

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Background

- Sampling from $\mathcal{D}_{\mathbb{Z},\sigma}$ for large σ: Base sampler with smaller deviation + Expander.
- Binary Sampling Algorithm [DDLL13]:
  - Base samplers: $x \hookleftarrow \mathcal{D}^{+}_{\mathbb{Z},\sigma_0}$, and $y \hookleftarrow \mathcal{U}(\{0,\ldots,k-1\})$, where $\sigma_0 = \sqrt{1/2\ln 2}$ and $k \in \mathbb{Z}^{+}$.
  - Expander: Compute $z = kx + y$. Use a Bernoulli sampler to reject z, with the acceptance bias: $p = \exp\left(-y(y+2kx)/2\sigma^2\right)$.
  - This algorithm will generate $z \hookleftarrow \mathcal{D}^{+}_{\mathbb{Z},k\sigma_0}$.
  - Side-channel attacks: [BHLY16, PBY17, EFGT17, BDE^{+}18].
  - Previous countermeasures:
    - Full-table access [PBY17, EFGT17].
    - Compute p by using a large look-up table (qTesla [BAA^{+}17]).

Renyi Divergence [Pre17]

- The Bernoulli sampler could fit in double precision (53 bits).
- Efficiency is independent of σ (relies on the number of samples instead).
- The $\exp(x)$ in C library is not constant-time (floating-point division).

Adapted from [Pre17], Lemma 3 and Eq. 4

For two distributions $\mathcal{P}$ and $\mathcal{Q}$ such that $\mathrm{Supp}(\mathcal{P}) = \mathrm{Supp}(\mathcal{Q})$, we have:

$$R_\alpha(\mathcal{P}||\mathcal{Q}) \leq 1 + \frac{\alpha \cdot (\Delta(\mathcal{P}||\mathcal{Q}))^2}{2},$$

when $\Delta(\mathcal{P}||\mathcal{Q}) \to 0$. In addition, for M independent samples, sampling from $\mathcal{P}$ will be λ-bit secure if $R_{2\lambda}(\mathcal{P}||\mathcal{Q}) \leq 1 + 1/(4M)$. Typically we have $M = m \cdot q_s$, where m is the dimension of the lattice, and q_s is the number of queries.

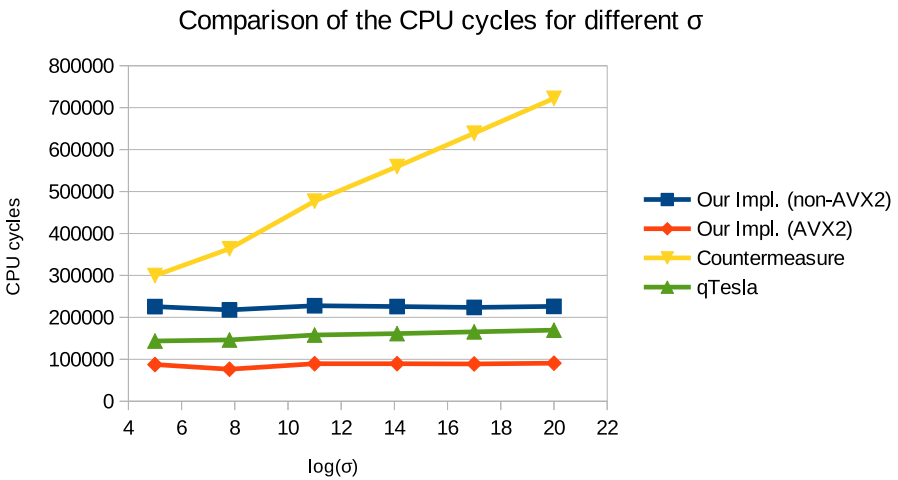
Proposed Techniques

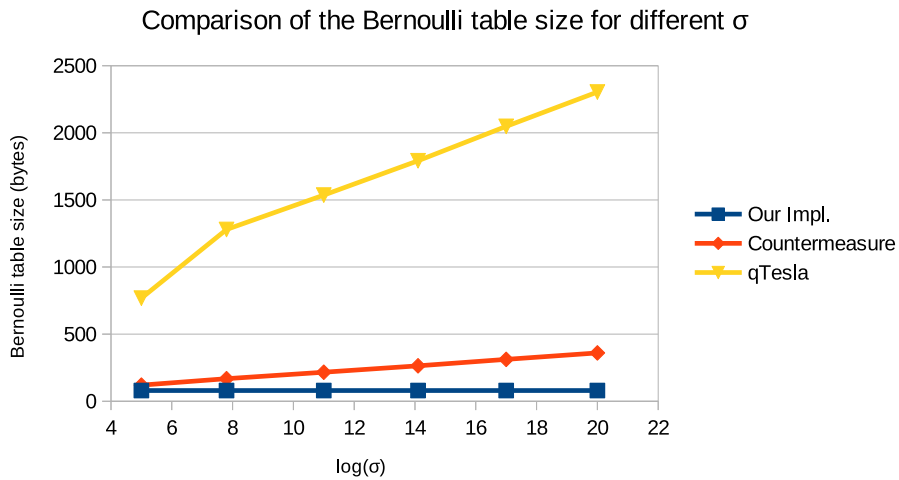
- Use a polynomial to estimate $\exp(x)$ with sufficient precision instead:
  1. Let $t = y(y + 2kx)$. Because $\sigma_0 = \sqrt{1/(2 \ln 2)}$ and $\sigma = k\sigma_0$, we rewrite the Bernoulli bias p as:

$$p = \exp\left(-t/2\sigma^2\right) = \exp\left(-\ln 2 \cdot t/k^2\right) = 2^{-t/k^2}.$$

  2. We adapt the techniques from [MBdD+10] to estimate $2^{-t/k^2}$:
     1. $2^{-t/k^2} = 2^{\lfloor -t/k^2 \rfloor + a} = 2^{\lfloor -t/k^2 \rfloor} \cdot 2^a$, $0 \leq a < 1$.
     2. Change the exponent of a double precision variable to get $2^{\lfloor -t/k^2 \rfloor}$.
     3. Use the Sollya tool [CJL10] to find a polynomial with sufficient precision for evaluating 2^a.

Efficiency Comparison





- $\lambda = 128,\ m = 1024,\ q_s = 2^{64}$.
- Use hardware AES-NI PRNG.

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