ZCZ: Achieving n-bit SPRP Security with a Minimal Number of Tweakable-block-cipher Calls

Ritam Bhaumik, Indian Statistical Institute, Kolkata

Eik List, Bauhaus-Universität Weimar, Weimar

Mridul Nandi, Indian Statistical Institute, Kolkata

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Introduction

Optimising SPRPs

SPRP: *Strong Pseudorandom Permutations* (indistinguishable under chosen-ciphertext attacks)

Common optimisation goals for SPRPs:

- Low implementation costs
- High provable-security guarantees
- High performance

Provable security of SPRPs

Adversary $\mathcal{A}$ making queries with at most σ blocks in all in CCA setting

Birthday Bound:
Distinguishing advantage $O\left(\frac{\sigma^2}{2^n}\right)$
(n: width of underlying primitive in bits, e.g., 128)

Here $\mathcal{A}$ needs $O\left(2^{n/2}\right)$ query-blocks to attack

Beyond Birthday Bound:
Number of query-blocks required of a higher order than $2^{n/2}$

Tweakable Block Ciphers

Blockcipher with additional public input

Dedicated Designs:
Deoxys-BC, Joltik-BC, Skinny

TBC-based MAC:
ZMAC [CRYPTO '17]

- Parallelisable
- Single-keyed
- Based on an internal hash function ZHash

Our Contributions

Theoretical:
Proof that 1.5 primitive calls per message block is close to optimal

Practical:
New TBC-based SPRP construction ZCZ:

- 1.5 TBC calls per message block
- Full *n*-bit provable security

ZCZ*: Extended version of ZCZ that can handle partial blocks

Preliminaries

Simple Random Sampling

Sample space: $\mathcal{S} = \{0, \ldots, N-1\}$
Sample: $(X_1, \ldots, X_q)$

With replacement (SRSWR):

- $X_1, \ldots, X_q$ independent
- For any $x_1, \ldots, x_q \in \mathcal{S}$, $\Pr[X_1 = x_1, \ldots, X_q = x_q] = 1/N^q$.

Without replacement (SRSWOR):

- $X_1, \ldots, X_q$ distinct, this is the only dependence
- For any $x_1, \ldots, x_q \in \mathcal{S}$,
  $\Pr[X_1 = x_1, \ldots, X_q = x_q] = (N-q)!/N!$ when $x_1, \ldots, x_q$ are distinct and 0 otherwise.

Collision Probabilities

Assume $X_1, \ldots, X_q$ is an SRSWR-sample from $\{0, \ldots, N-1\}$

Single collision:

$$\alpha_0 + \alpha_1 X_1 + \ldots + \alpha_q X_q = 0$$

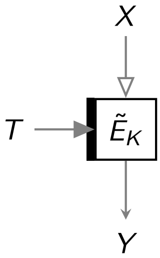
If $\alpha_i \neq 0$ for any $i \in \{1, \ldots, q\}$, the probability is $1/N$.

Double collision:

$$\alpha_0 + \alpha_1 X_1 + \ldots + \alpha_q X_q = 0,$$
$$\beta_0 + \beta_1 X_1 + \ldots + \beta_q X_q = 0.$$

If $\alpha_i \beta_j \neq \beta_i \alpha_j$ for any $i, j \in \{1, \ldots, k\}$, the probability is $1/N^2$.

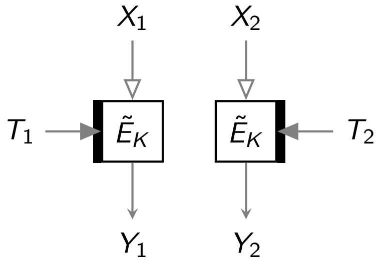
Tweakable Blockciphers



$$\tilde{E} : \mathcal{K} \times \mathcal{T} \times \mathcal{B} \longrightarrow \mathcal{B}$$

$\mathcal{K}$: Key space, $\mathcal{T}$: Tweak space, $\mathcal{B}$: Block space
For fixed K and T, $\tilde{E}_K(T, \cdot)$ is injective

Constraints



$$(X_1, T_1) = (X_2, T_2) \implies Y_1 = Y_2$$
$$(Y_1, T_1) = (Y_2, T_2) \implies X_1 = X_2$$

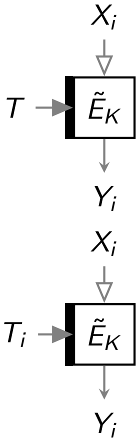
[No constraints when $T_1 \neq T_2$]

Ideal Tweakable Blockciphers

For random $K \in \mathcal{K}$:

- For fixed $T \in \mathcal{T}$, for distinct $X_1, \dots, X_q \in \mathcal{B}$, $(Y_1, \dots, Y_q)$ should form an SRSWOR-sample from $\mathcal{B}$.

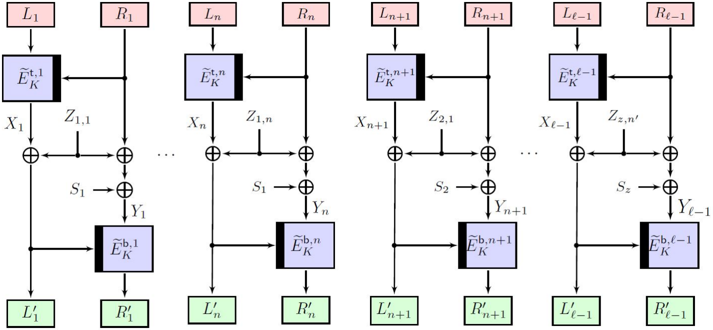
- For distinct $T_1, \dots, T_q \in \mathcal{T}$, for any $X_1, \dots, X_q \in \mathcal{B}$, $(Y_1, \dots, Y_q)$ should form an SRSWR-sample from $\mathcal{B}$.



Construction

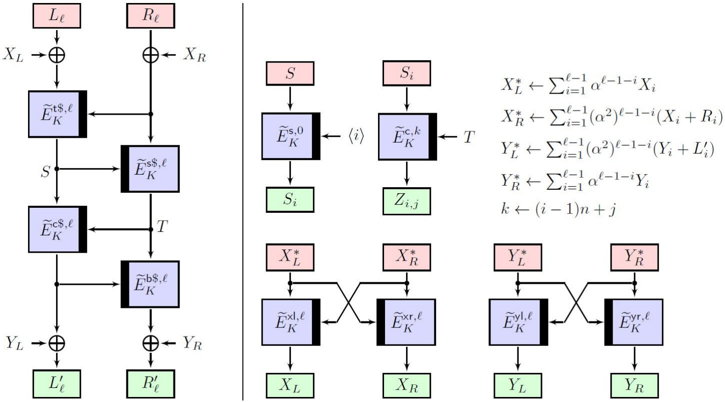
The ZCZ Encryption Scheme

(a) The first $\ell - 1$ diblocks ($Z_{i,j}$, S_i mixing variables):



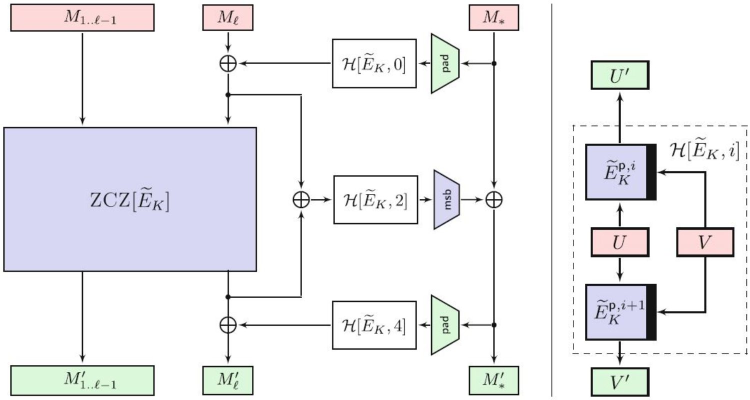
The ZCZ Encryption Scheme

(b) The final diblock and the mixing layer:

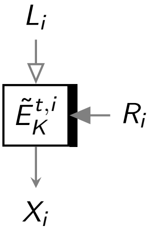


The ZCZ* Encryption Scheme

Version of ZCZ that can handle partial blocks:



Breaking it down



Top Layer

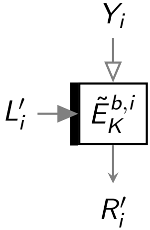
$$\{(X_i, R_i) \mid 1 \leq i \leq \ell - 1\}, (L_\ell, R_\ell)$$

$$\downarrow$$

$$\boxed{\text{Mixing Layer}}$$

$$\downarrow$$

$$\{(L'_i, Y_i) \mid 1 \leq i \leq \ell - 1\}, (L'_\ell, R'_\ell)$$



Bottom Layer

(The different tweakable blockciphers used are obtained from a single key using standard domain separation techniques)

Proof Approach

Main Tool

Coefficient H Technique: For real oracle $\mathcal{O}_1$ and ideal oracle $\mathcal{O}_0$, consider the conditions below:

- *Condition 1:* The probability of certain **bad** events occurring under the ideal oracle is bounded above by ϵ;
- *Condition 2:* When a bad event has not occurred, a transcript is at least as probable under $\mathcal{O}_1$ as under $\mathcal{O}_0$.

When these conditions hold, the advantage of an adversary distinguishing between $\mathcal{O}_0$ and $\mathcal{O}_1$ is bounded above by ϵ.

Oracles

Real Oracle:

- Uses ZCZ to answer queries;
- At the end reveals all internal inputs and outputs to $\tilde{E}_K$.

Ideal Oracle:

- Uses an ideal random wide permutation to answer queries;
- At the end *samples* all internal inputs and outputs to $\tilde{E}_K$;
- Sampling is first done over a **basis**;
- This sample is then **extended** to the other inputs and outputs.

Bad Events: Main Idea

Broad strategy:

- Ban (tweak, input) collisions;
- Ban (tweak, output) collisions.

We need to ensure that each bad event is a double collision.

Bounding Bad Probabilities

Approach:

- Fix all parameters;
- Identify underlying double collision;
- Check conditions for $1/N^2$ bound (where $N = 2^n$).
- Count over parameter choices;
- Bound using union bound.

Final bound obtained on distinguishing advantage: $\frac{21\sigma^2}{2^{2n}}$

Thank you for your attention

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