On the Statistical Leak of the GGH13 Multilinear Map and some Variants

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What is this talk about?

Objective: Analyse the statistical leak of the GGH13 multilinear map

GGH13: Garg, Gentry and Halevi. Candidate multilinear maps from ideal lattices, Eurocrypt.

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  - For 4 different variants of the GGH13 map
- Proposition of a countermeasure

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Cryptographic multilinear map

Definition: κ asymmetric multilinear map

Different levels of encodings, corresponding to subsets of $\{1, \dots, \kappa\}$.
Denote by $\mathsf{Enc}(a, S)$ a level-S encoding of the message a, for $S \subseteq \{1, \dots, \kappa\} =: [\kappa]$.

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Functionality:

Addition: $\mathrm{Add}(\mathrm{Enc}(a_1, S), \mathrm{Enc}(a_2, S)) = \mathrm{Enc}(a_1 + a_2, S)$

Multiplication: $\mathrm{Mult}(\mathrm{Enc}(a_1, S_1), \mathrm{Enc}(a_2, S_2)) = \mathrm{Enc}(a_1 \cdot a_2, S_1 \cup S_2)$ if $S_1 \cap S_2 = \emptyset$

Zero-test: $\text{Zero-test}(\mathrm{Enc}(a, [\kappa])) = \text{True iff } a = 0$

Security: multiple security definitions

Mmap: applications and candidates

Applications:

- One-round key-exchange between $\kappa + 1$ users (generalization of pairings)
- Attribute based encryption, witness encryption, ...
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Three main candidates

GGH13, CLT13, GGH15

GGH13: Garg, Gentry and Halevi (Eurocrypt 2013)
CLT13: Coron, Lepoint, Tibouchi (Crypto 2013)
GGH15: Gentry, Gorbunov, Halevi (TCC 2015)

Previous attacks on GGH13 map

Zeroizing attacks

Theorem [Hu and Jia, Eurocrypt'16]

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Statistical attacks

- mentioned in [GGH13]
  - 2 sampling methods proposed

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In this talk

The leak we analyse is the variance of the post-zero-tested elements

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We define our own setting

- inspired by iO
- but simpler
- secure in the weak multilinear map model
  - no "simple" zeroizing attacks

Contribution (2)

- We consider 4 different sampling procedures for the encodings:
  - 2 from [GGH13]
  - 2 from [DGG+18]

[DGG+18] Döttling, Garg, Gupta, Miao, and Mukherjee. Obfuscation from Low Noise Multilinear Maps, Indocrypt.

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Sampling method	leakage related to secret elements	full attack?
Simplistic [GGH13]	yes	yes for some params
Exponential [GGH13]	yes	no
Conservative [DGG+18]	yes	no
Aggressive [DGG+18]	yes	no

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Compensation (this work)	no	no

- We propose a countermeasure ⇒ Compensation method
  - In **this** simple setting
  - Almost as efficient as the simplistic method

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Outline of the talk

The GGH13 multilinear map

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- Define $R = \mathbb{Z}[X]/(X^n + 1)$ with $n = 2^k$
- Sample g a "small" element in R
  $\Rightarrow$ the plaintext space is $\mathcal{P} = R/\langle g \rangle$
- Sample q a "large" integer
  $\Rightarrow$ the encoding space is $R_q = R/(qR) = \mathbb{Z}_q[X]/(X^n + 1)$

Notation

We write $[r]_q$ for the elements in R_q

The GGH13 multilinear map: encodings

- Sample $z_1, \dots, z_\kappa$ uniformly in R_q
- **Encoding:** An encoding of a at level $S \subseteq \{1, \dots, \kappa\}$ is

$$u = \left[\frac{\widetilde{a}}{\prod_{i \in S} z_i} \right]_q$$

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Addition and multiplication

Addition:

$$\left[\frac{a_1 + r_1 g}{\prod_{i \in S} z_i} \right]_q + \left[\frac{a_2 + r_2 g}{\prod_{i \in S} z_i} \right]_q = \left[\frac{a_1 + a_2 + r' g}{\prod_{i \in S} z_i} \right]_q$$

Multiplication:

$$\left[\frac{a_1 + r_1 g}{\prod_{i \in S_1} z_i} \right]_q \cdot \left[\frac{a_2 + r_2 g}{\prod_{i \in S_2} z_i} \right]_q = \left[\frac{a_1 \cdot a_2 + r' g}{\prod_{i \in S_1 \cup S_2} z_i} \right]_q \quad (\text{if } S_1 \cap S_2 = \emptyset)$$

The GGH13 multilinear map: zero-test

- Sample h in R of the order of $q^{1/2}$
- Let $z^* = \prod_{i=1}^{\kappa} z_i$
- Define

$$p_{zt} = [z^* h g^{-1}]_q$$

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Zero-test

To test if $u = [c/z^*]_q$ is an encoding of zero (i.e. $c = 0 \mod g$), compute

$$[u \cdot p_{zt}]_q = [chg^{-1}]_q$$

This is small iff c is a small multiple of g.

Remark: If $c = 0 \mod g$, then $\boxed{[u \cdot p_{zt}]_q = ch/g \text{ over } R}$

Outline of the talk

1 The GGH13 Map

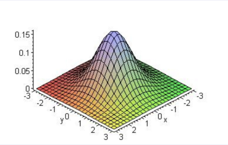
2 Statistical Leak

Statistical background (1)

Definitions

A distribution is said **centered** if its mean is zero.
A distribution is said **isotropic** if no direction is privileged.

Example



Notation: We write in red the centered isotropic variables

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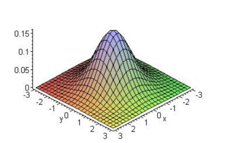

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Gaussian distribution

We write $D_{L,\sigma}$ the discrete Gaussian distribution centered in 0 and of variance parameter σ^2 over the lattice L

$D_{L,\sigma}$ is a centered isotropic distribution

Statistical background (2)

Definitions / Notation

- For $r \in R$, we denote $A(r) = r\bar{r}$ the **auto-correlation** of r, where $\bar{r}$ is the complex conjugate of r when seen in $\mathbb{C}$
- The **variance** of a centered variable r is $\text{Var}(r) := \mathbb{E}(r\bar{r})$

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Proposition: If r is centered and isotropic then

$$\mathbb{E}(r) = 0$$
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In this talk, assume $\mathrm{Var}(r) = 1$

Statistical leak

Recall

If $u = [c/z^*]_q$ with $c = 0 \bmod g$, then

$$[u \cdot p_{zt}]_q = c \cdot h/g \in R$$

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Idea: h/g is fixed but c is a random variable

$$\boxed{\mathrm{Var}(c \cdot h/g) = \mathrm{Var}(c) \cdot A(h/g)}$$

We can approximate it with many samples

Simple setting (simplified)

- For all $1 \leq i \leq \kappa$, we get
  - $[\frac{\widetilde{a_i}}{z_i}]_q$ with $\widetilde{a_i} = a_i \bmod g$
  - $[\frac{\widetilde{b_i}}{\prod_{j \neq i} z_j}]_q$ with $\widetilde{b_i} = b_i \bmod g$
- such that $a_i b_i = 0$

1	2	3	$\cdots$	i	$\cdots$	κ

a_i

b_i

$a_i b_i = 0$

Leak in the simple setting

We get encodings of zero:

$$u_i = \left[\frac{\widetilde{a}_i}{z_i}\right]_q \cdot \left[\frac{\widetilde{b}_i}{\prod_{j \neq i} z_j}\right]_q = \left[\frac{\widetilde{a}_i \widetilde{b}_i}{z^*}\right]_q$$

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After zero-test:

$$(\widetilde{a_i} \cdot \widetilde{b_i}) \cdot h/g \in R$$

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Variance:

$$\mathrm{Var}(\widetilde{a}_i \cdot \widetilde{b}_i) \cdot A(h/g) = \mathrm{Var}(\widetilde{a}_i) \cdot \mathrm{Var}(\widetilde{b}_i) \cdot A(h/g)$$

Leakage for the two methods of [GGH13]

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The leakage is

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$\widetilde{a_i} \leftarrow D_{a_i+gR,\sigma}$

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$$\widetilde{a_i} = \widehat{a_i} \cdot z_i$$
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$$\text{for } \widehat{a_i} \leftarrow D_{(a_i+gR)/z_i,\ \sigma}$$
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Leakage:

$$A(z_i) \cdot A(\prod_{j \neq i} z_j) \cdot A(h/g)$$
$$= A(z^* h/g)$$

Countermeasure

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Leakage:

$$A(\sqrt{g/h}) \cdot A(\sqrt{g/h}) \cdot A(h/g)$$
$$= 1$$

Remark: more efficient than other methods (except simplistic)

What to do with the leakage

	Simplistic method	Exponential method
Leakage	$\approx A(h/g)$	$\approx A(z^* h/g)$

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Remark: does not work for $A(z^* h/g)$

Conclusion

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