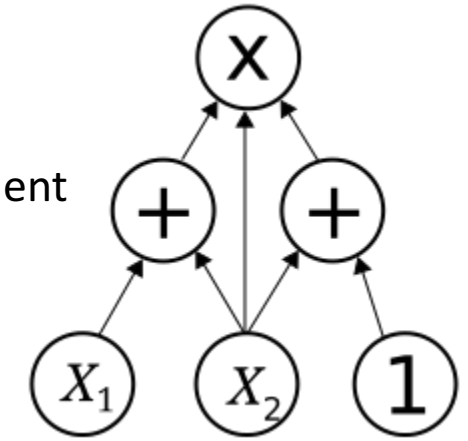
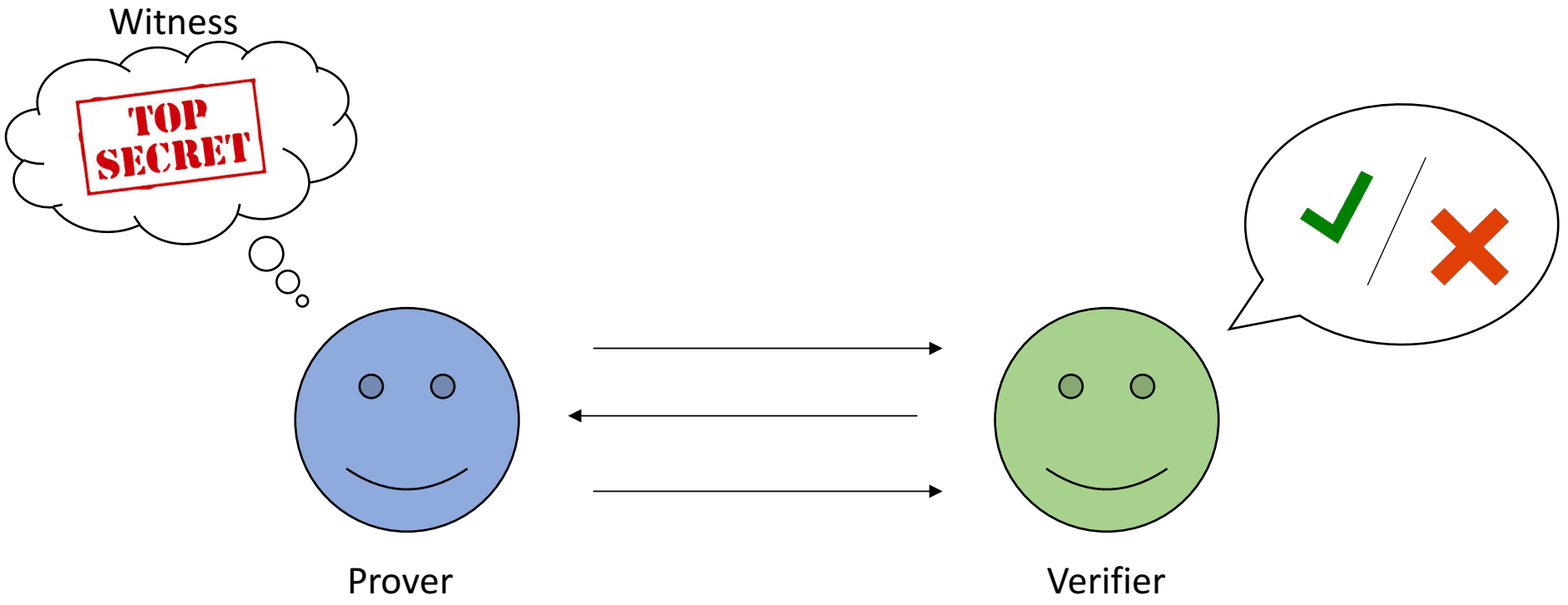
Arya:
Nearly Linear-Time Zero-Knowledge Proofs for Correct Program Execution

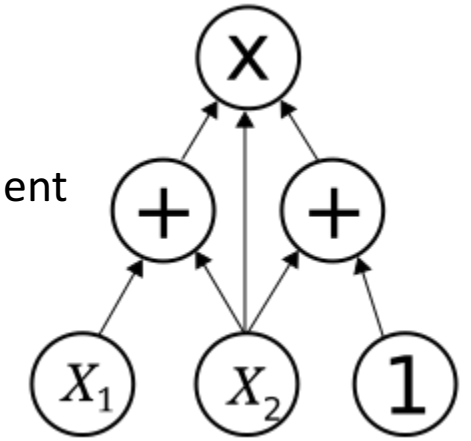
Jonathan Bootle, Andrea Cerulli, Jens Groth, Sune Jakobsen, Mary Maller

Zero-Knowledge Proofs for Correct Program Execution

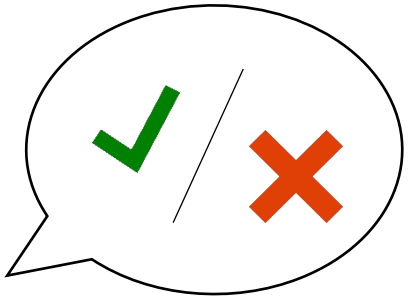


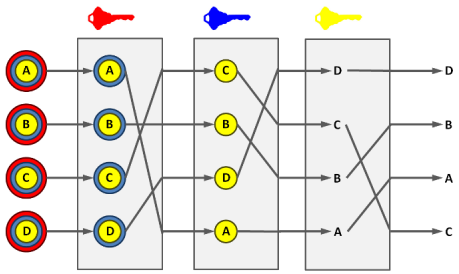


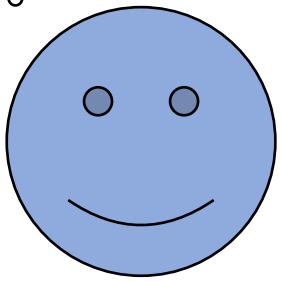
Zero-Knowledge Proofs for Correct Program Execution



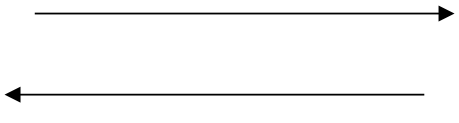
Witness



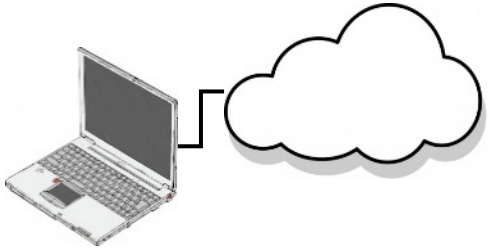




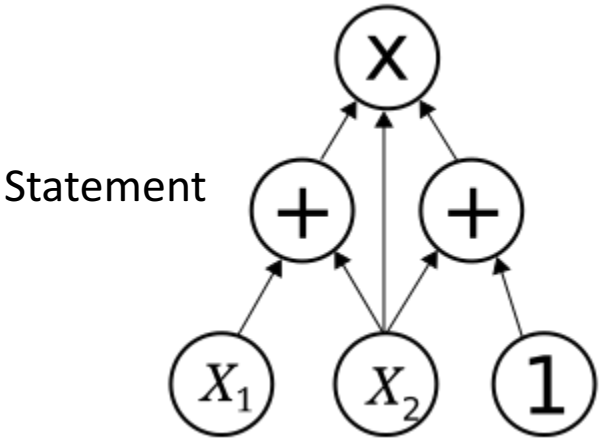
Prover

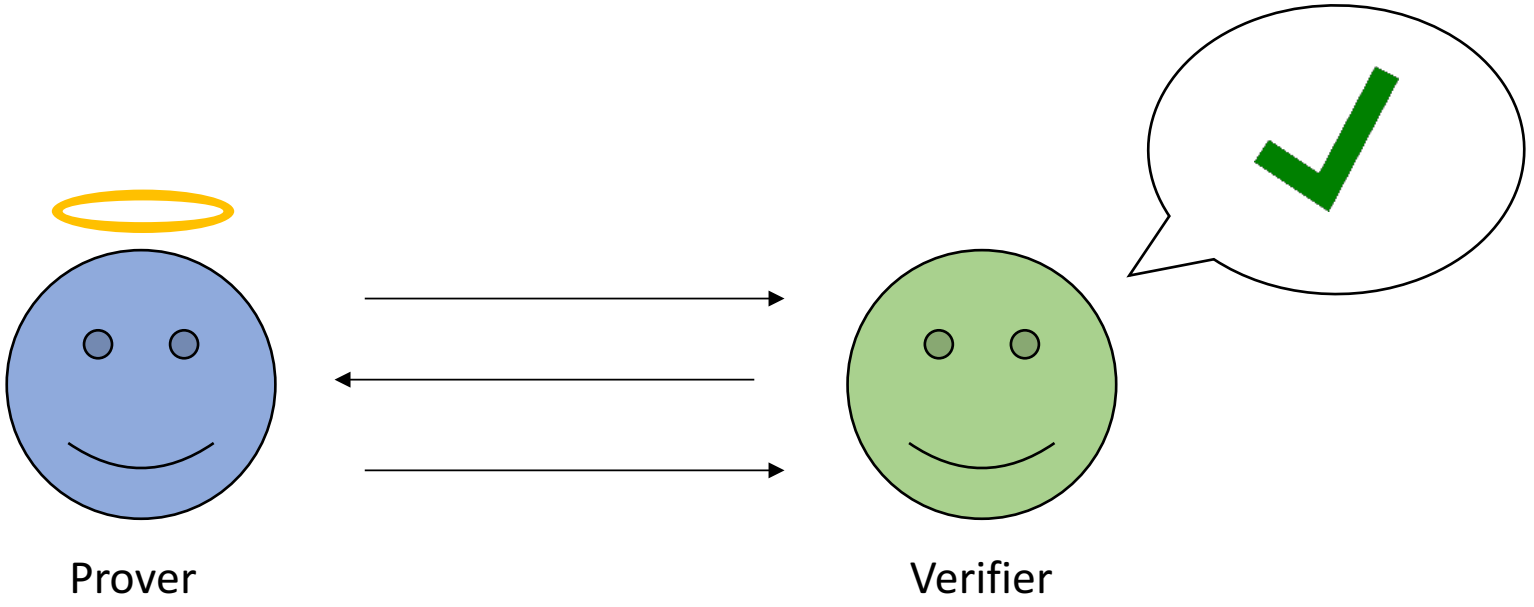


Verifier



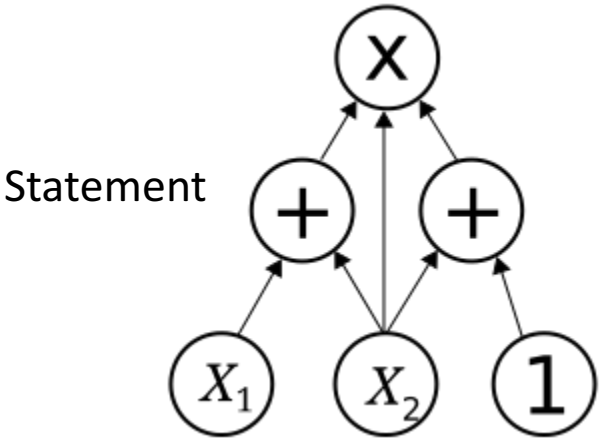
Zero-Knowledge Proofs for Correct Program Execution

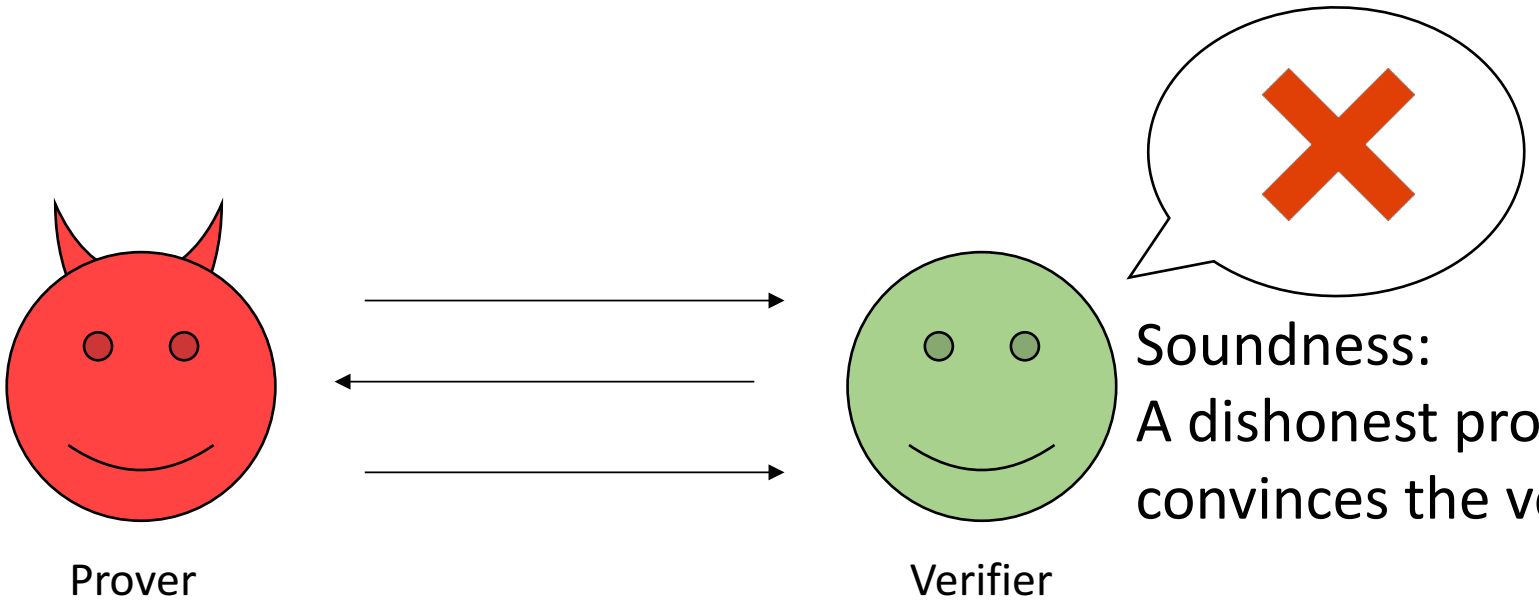




Completeness:
An honest prover convinces the verifier.

Zero-Knowledge Proofs for Correct Program Execution



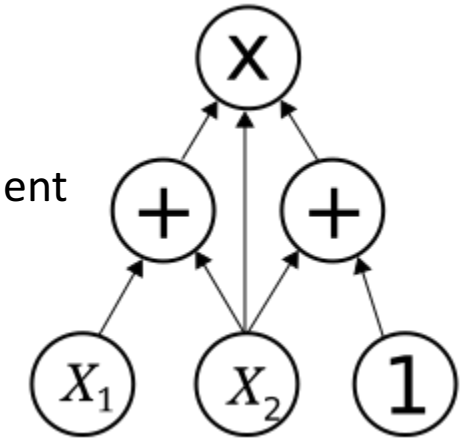


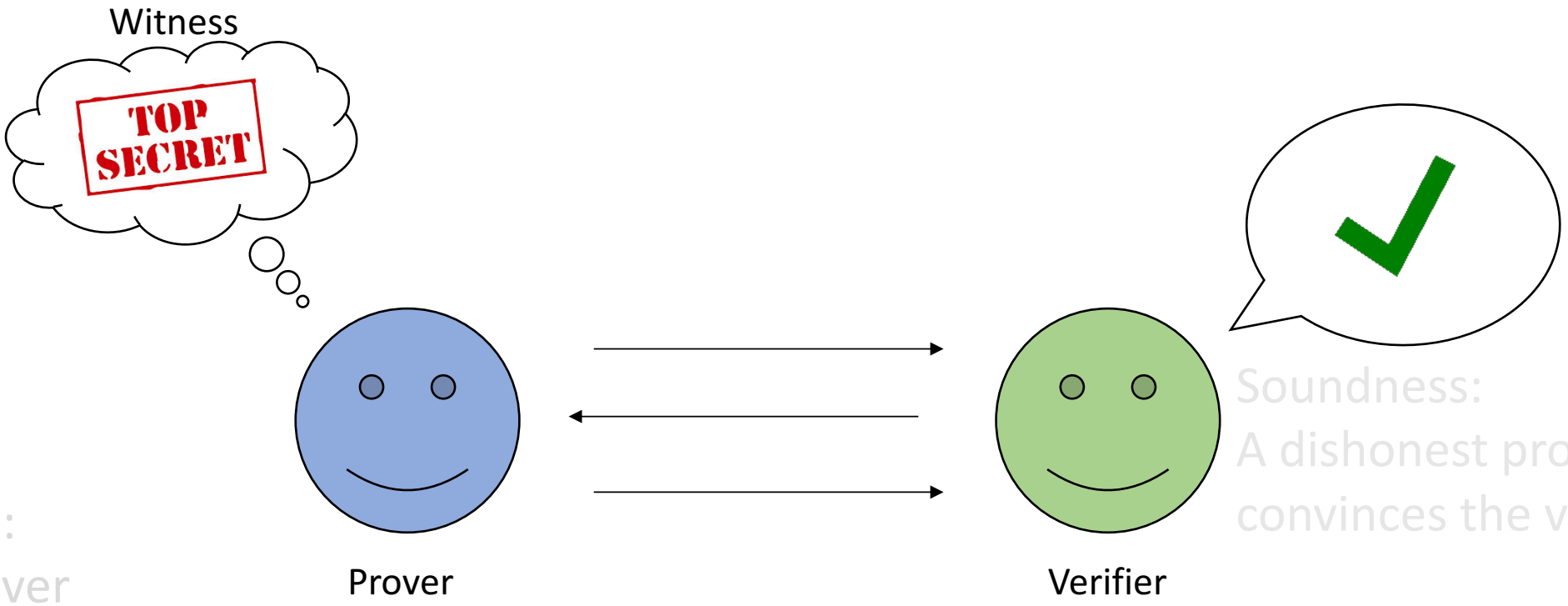
Soundness:
A dishonest prover never convinces the verifier.

Computational guarantee
-> argument

Completeness:
An honest prover convinces the verifier.

Zero-Knowledge Proofs for Correct Program Execution





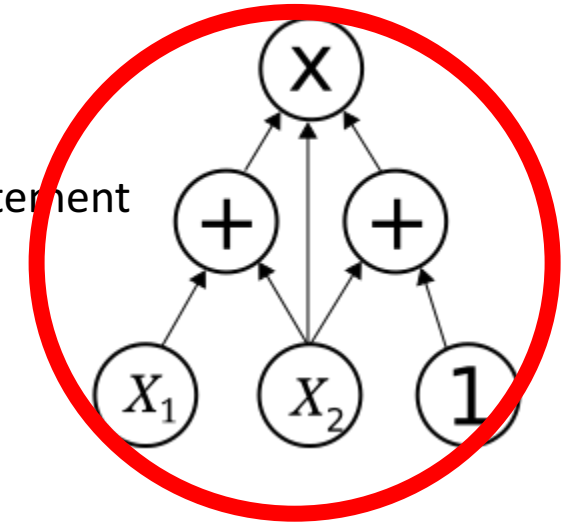
Completeness:
An honest prover convinces the verifier.

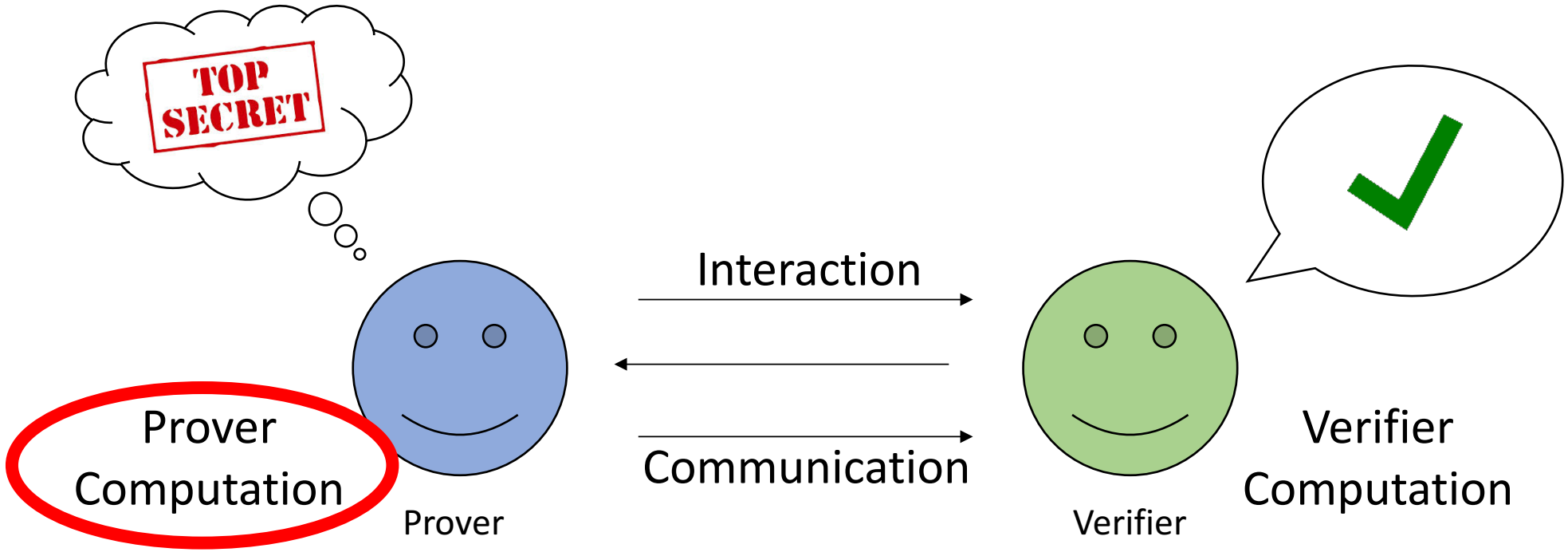
Soundness:
A dishonest prover never convinces the verifier.

Computational guarantee
-> argument

Zero-knowledge:
Nothing but the truth of the statement is revealed.

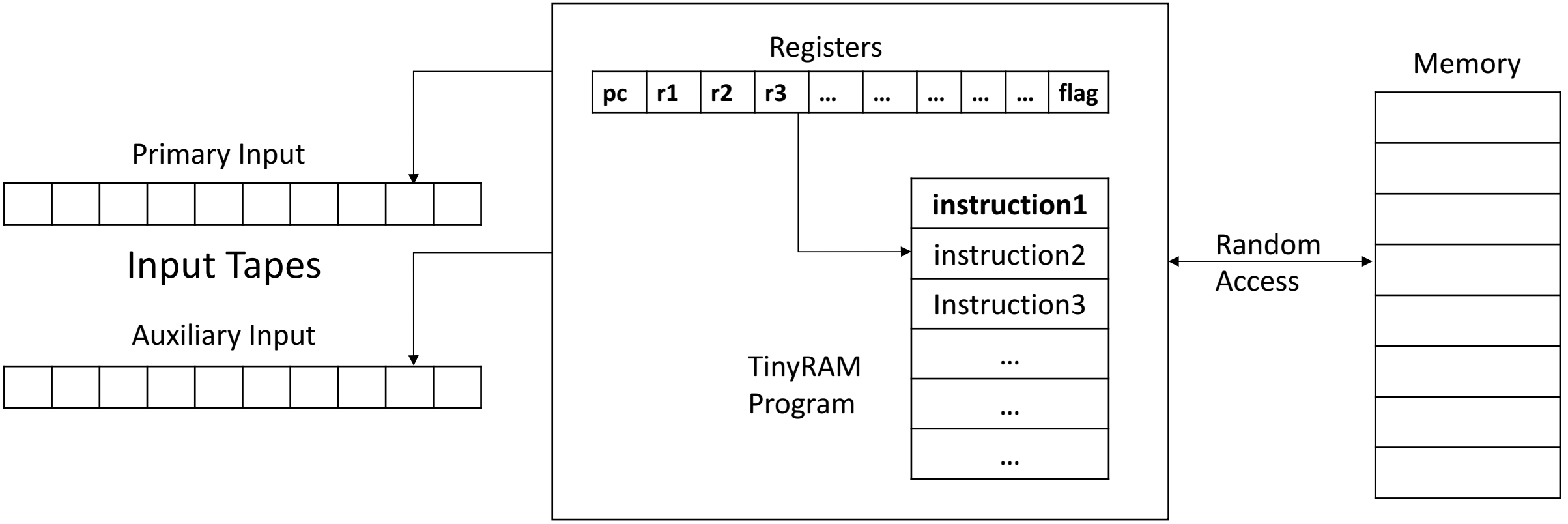
Zero-Knowledge Proofs for Correct Program Execution





Zero-Knowledge Proofs for

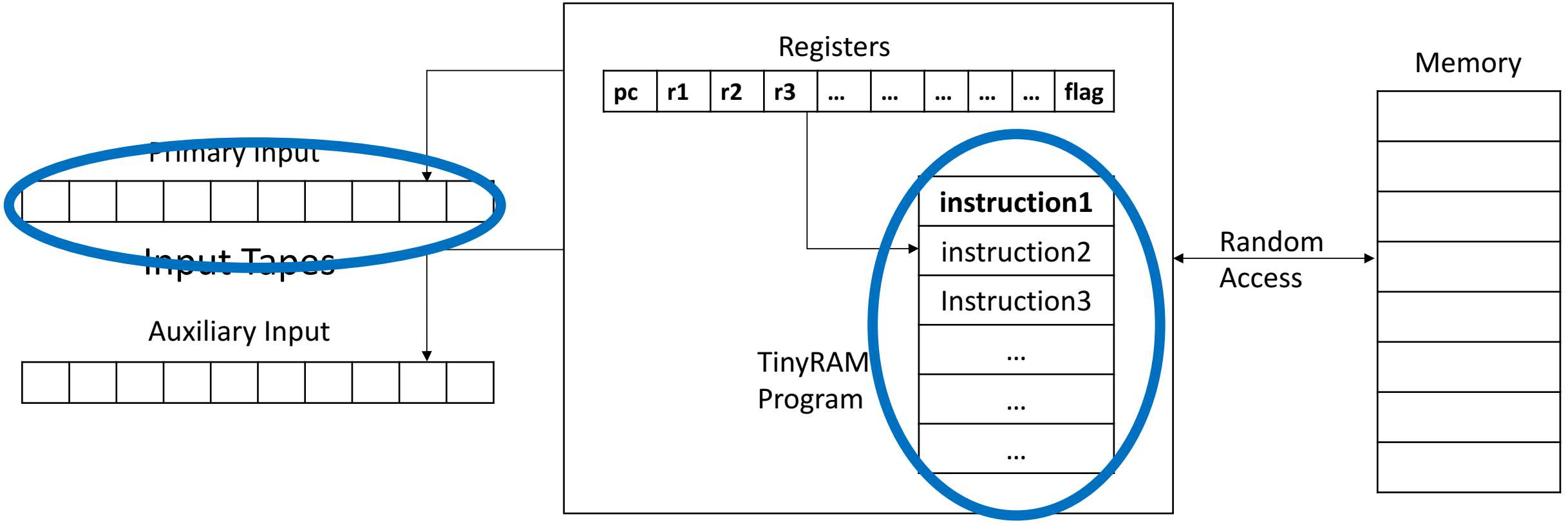
Correct Program Execution



TinyRAM

Instructions include ADD, MULT, XOR, AND,…

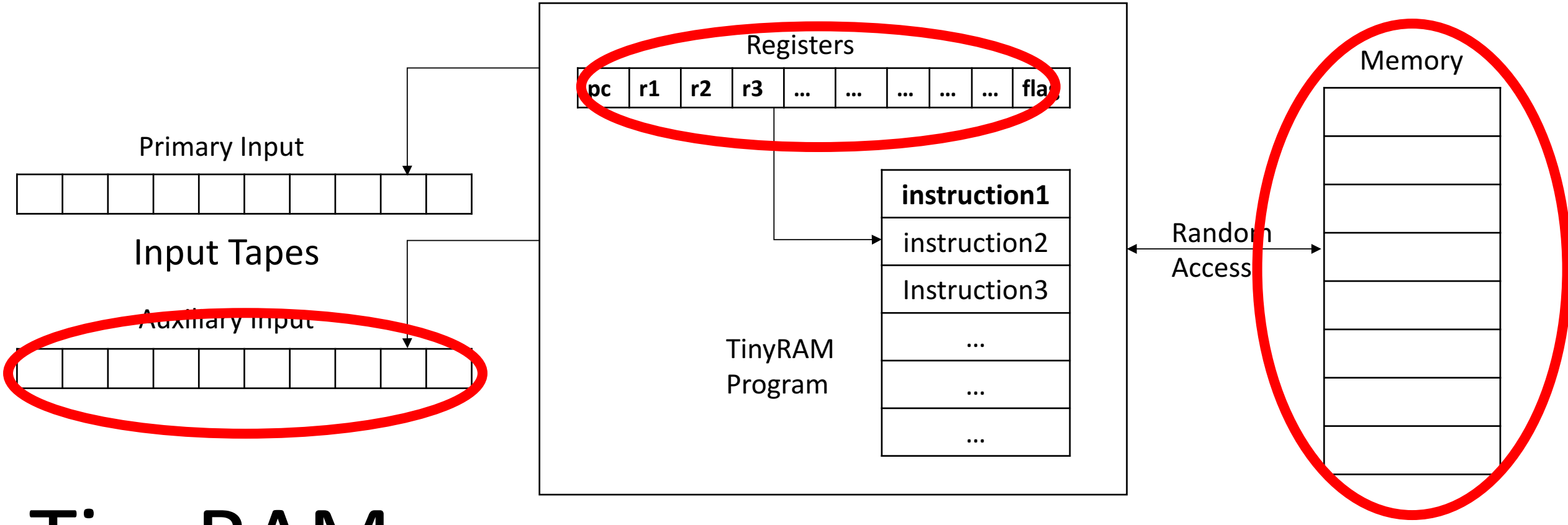
Zero-Knowledge Proofs for Correct Program Execution



TinyRAM

Public Values <-> Statement

Zero-Knowledge Proofs for Correct Program Execution



TinyRAM

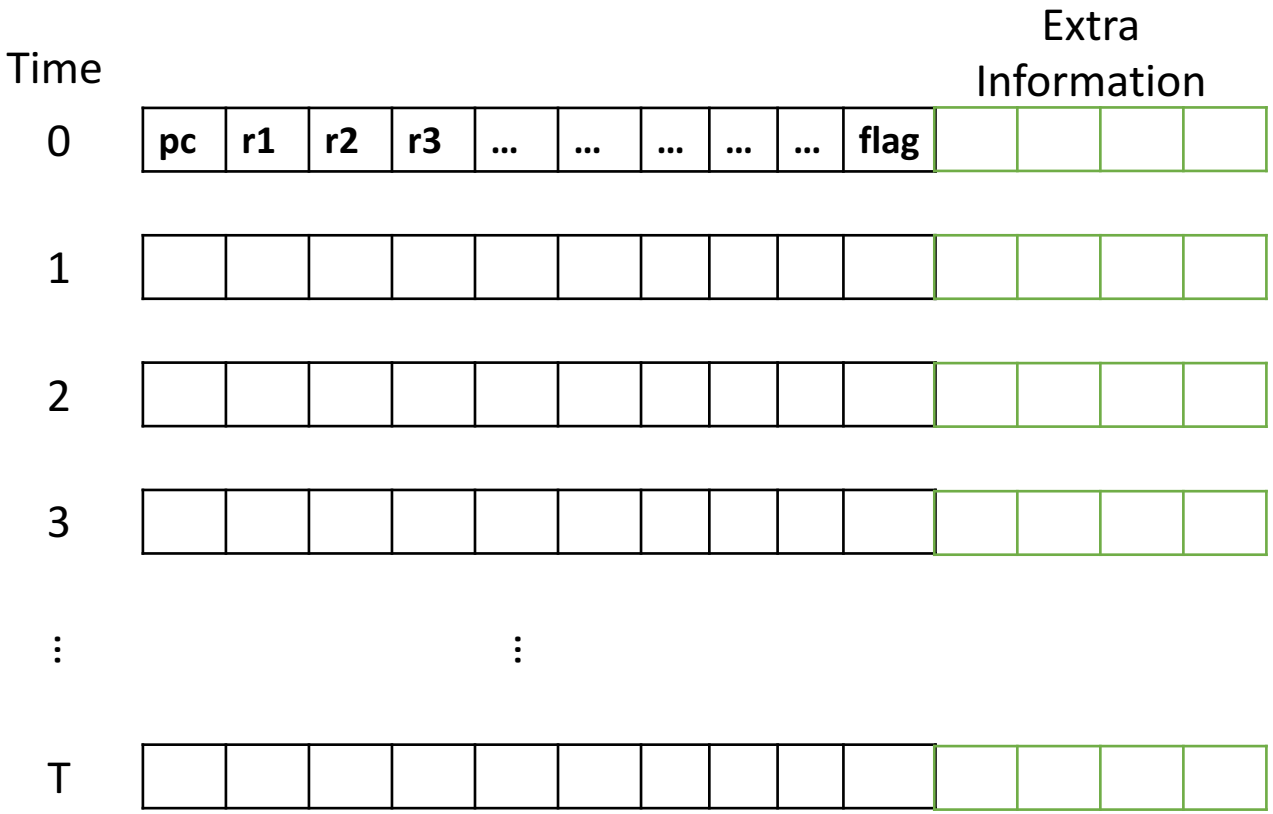
Private Values <-> Prover's Witness

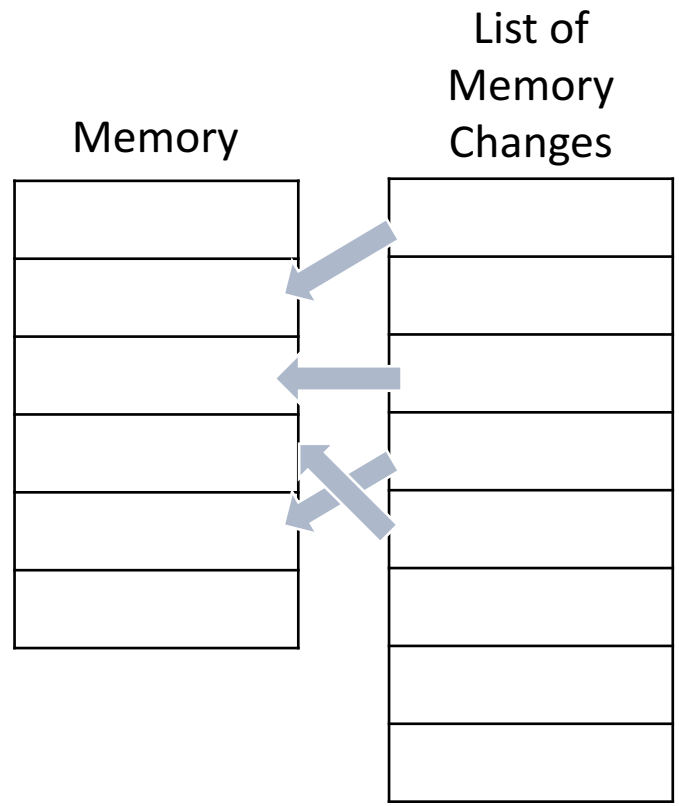
Zero-Knowledge Proofs for Correct Program Execution

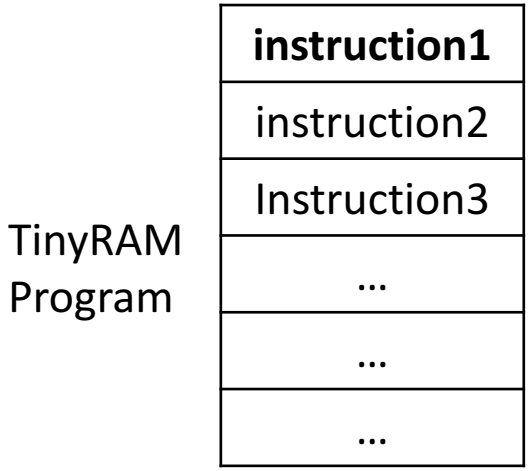
Goal:

Zero-knowledge proof for correct TinyRAM execution with low prover overhead

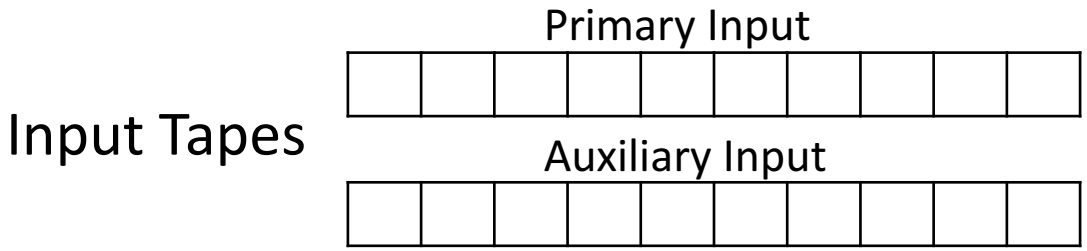
Execution Trace



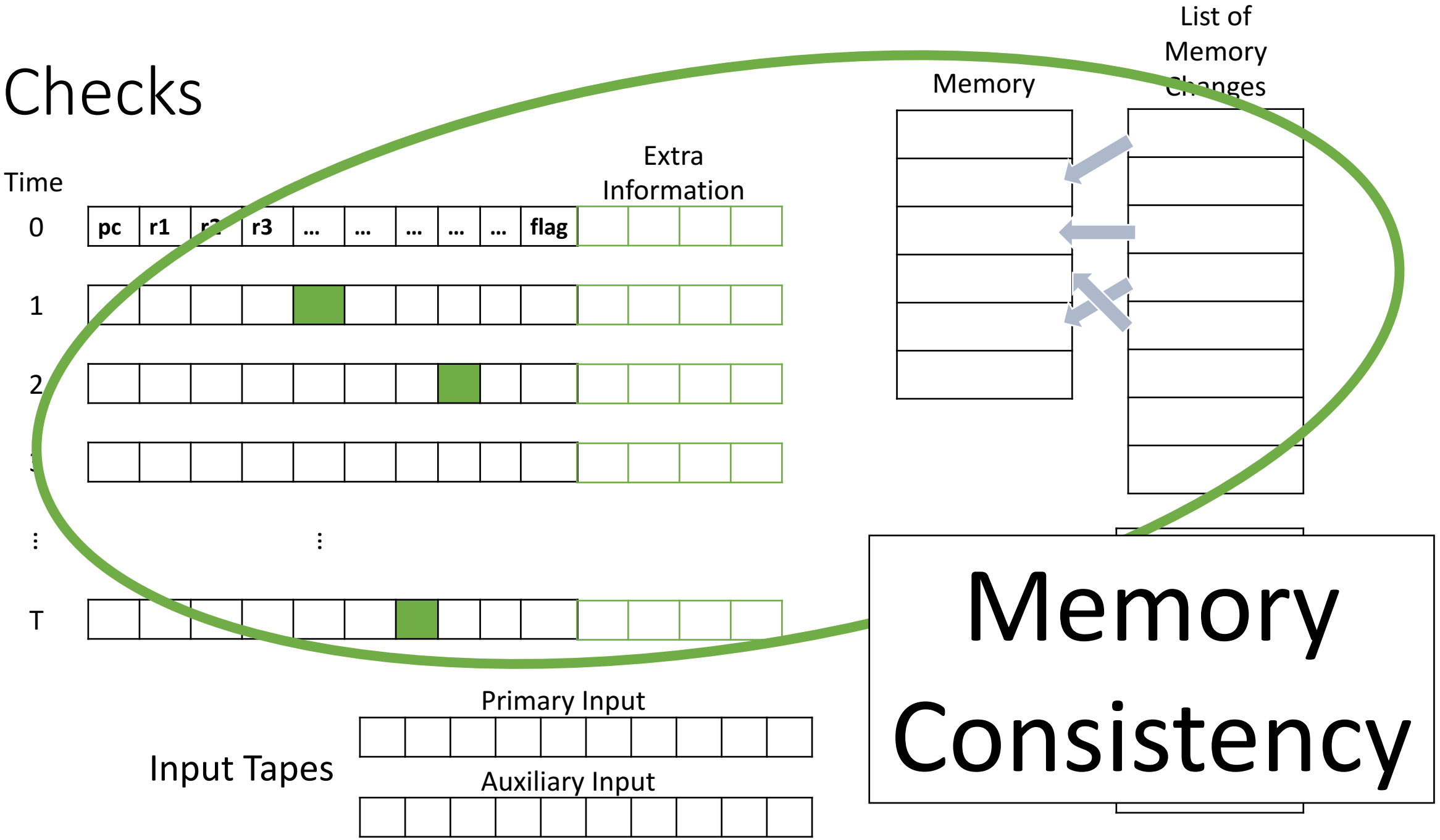




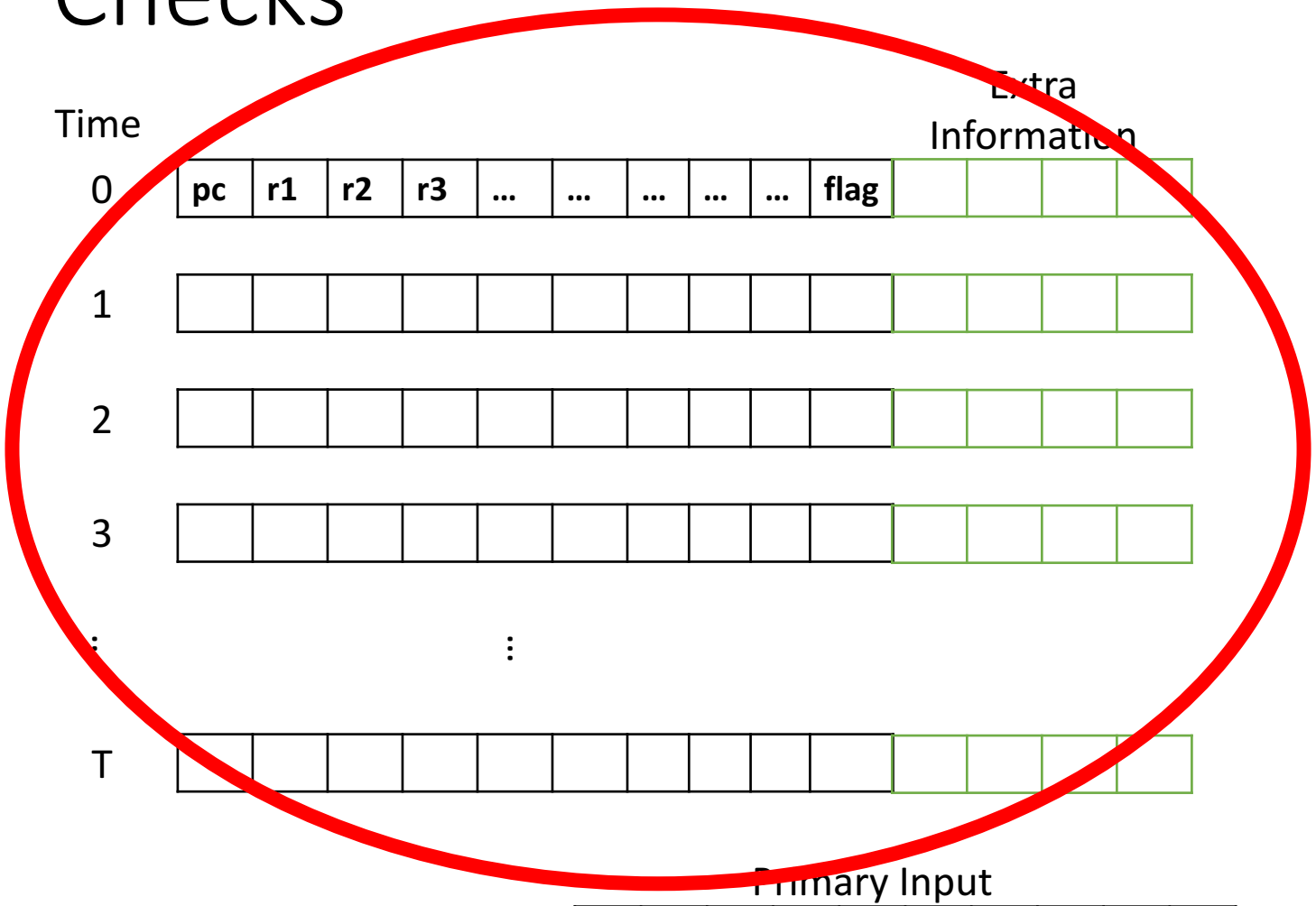
TinyRAM Program

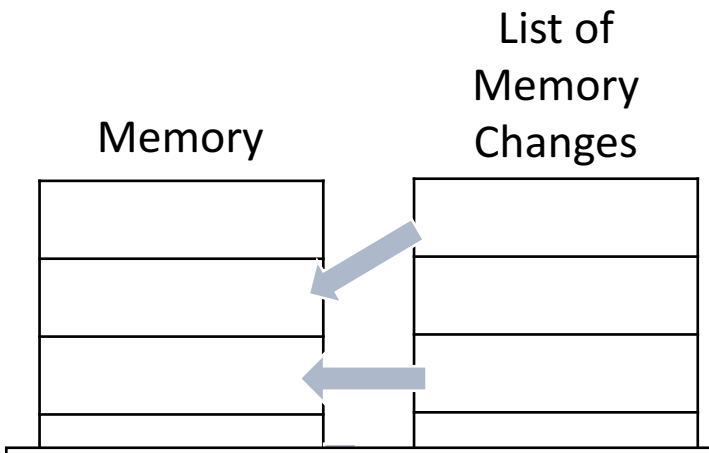


Checks

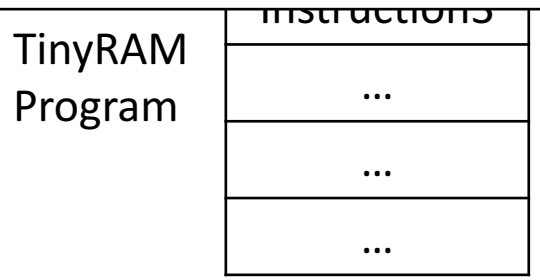

Time
Extra Information
pc
r1
r3
flag
0
1
2
T
Memory
List of Memory Changes
Memory Consistency
Input Tapes
Primary Input
Auxiliary Input

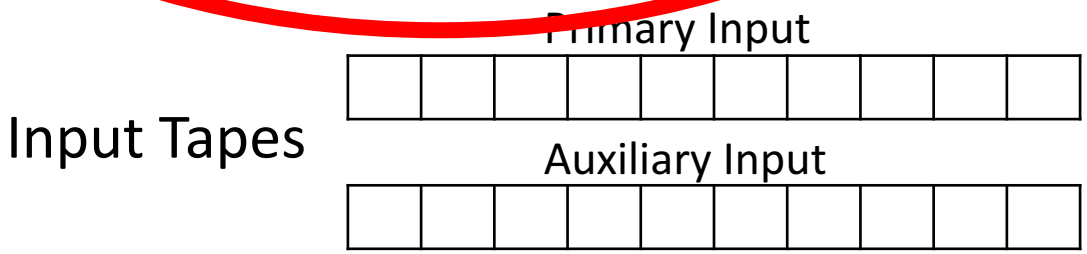
Checks



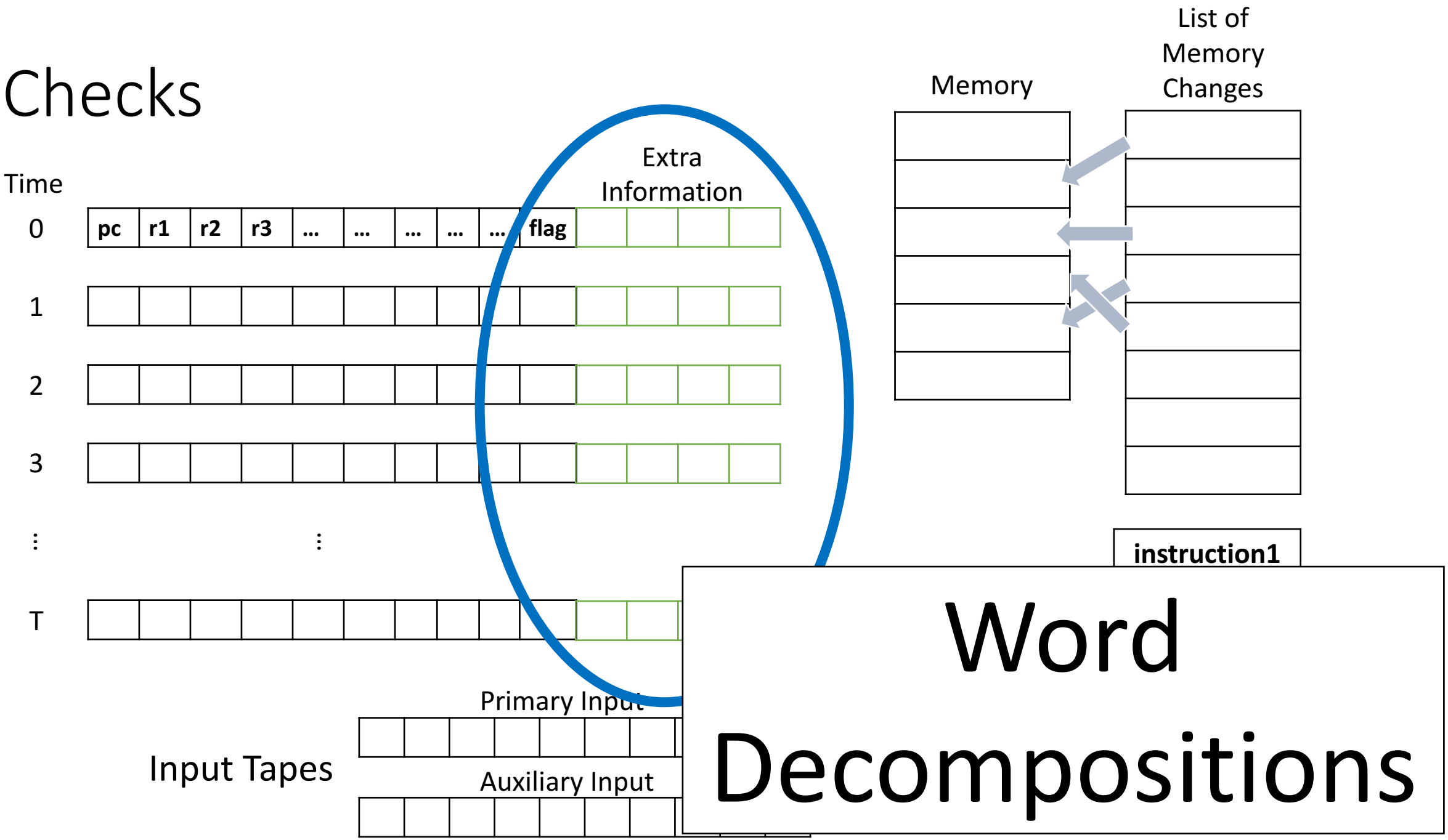


Correct Instruction Execution





Checks


Time
0
1
2
3
T
pc
r1
r2
r3
...
...
...
...
...
flag
Extra Information
Memory
List of Memory Changes
instruction1
Primary Input
Input Tapes
Auxiliary Input
Word Decompositions

Proving Correct Program Execution

Sources of Overhead

- Large fields and large cyclic groups
- Permutation networks for checking memory
- Large circuits for bitwise operations

Our Solutions

- Use hash-based proof system over any field
- Alternative approach to checking permutations
- Word decomposition technique gives constant-size circuits

Results

Work	Prover Complexity	Verifier Complexity	Communication	Rounds	Assumption
BCTV14	$\Omega(T(\log T)^2)$	$\omega(L + \lvert v \rvert)$	$\omega(1)$	1	KoE
This Work	$O(\alpha T)$	$poly(\lambda)(\sqrt{T} + L + \lvert v \rvert)$	$poly(\lambda)(\sqrt{T} + L)$	$O(\log \log T)$	LT-CRHF

But what is α?

Security $2^{-\omega(\log \lambda)}$

Program Length L

Runtime Bound T

Public Input V

Overview

TinyRAM Execution Trace

Algebraic Constraints

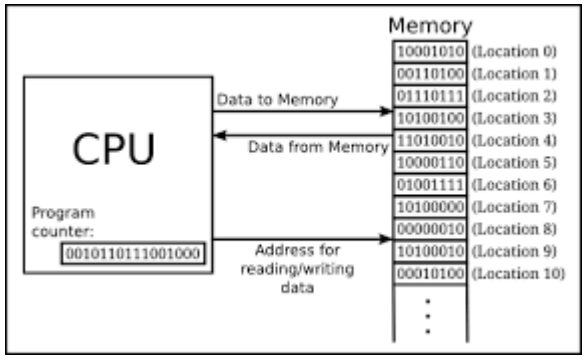
Polynomials

Ideal Protocols

Standard Protocols

Prior work: Linear-Time Zero-Knowledge Proofs

Arithmetising TinyRAM



Main Contributions

Zero-Knowledge Look-Up Arguments and More

Table of baby-name data (baby-2010.csv)

name	rank	gender	year
Jacob	1	boy	2010
Isabella	1	girl	2010
Ethan	2	boy	2010
Sophia	2	girl	2010
Michael	3	boy	2010

Field names

One row (4 fields)

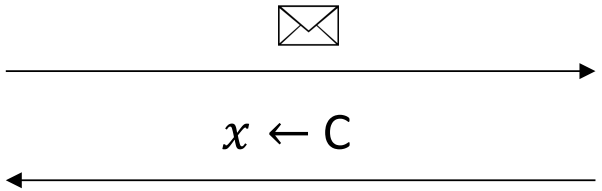
2000 rows all told

Ideal Linear Commitment Protocols

P

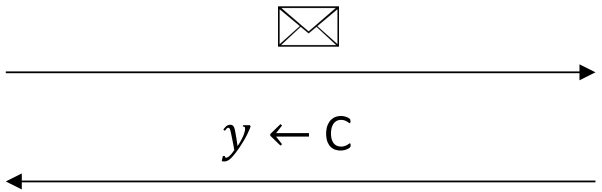
V

Commit to vectors

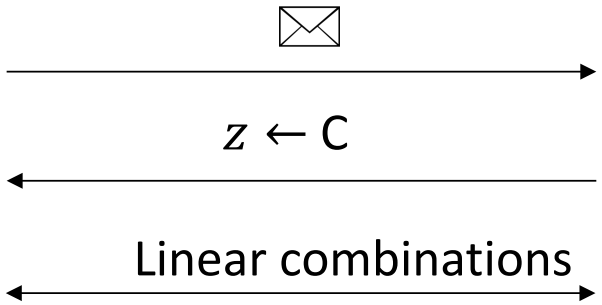


Commit to vectors

Send random challenges



⋮ ⋮



Compute linear combinations

Check linear combinations against commitments

Ideal Linear Commitment Protocols

P

V

Commit to execution trace

$x \leftarrow \mathsf{C}$

Commit to vectors

Send random challenges

$y \leftarrow \mathsf{C}$

⋮ ⋮

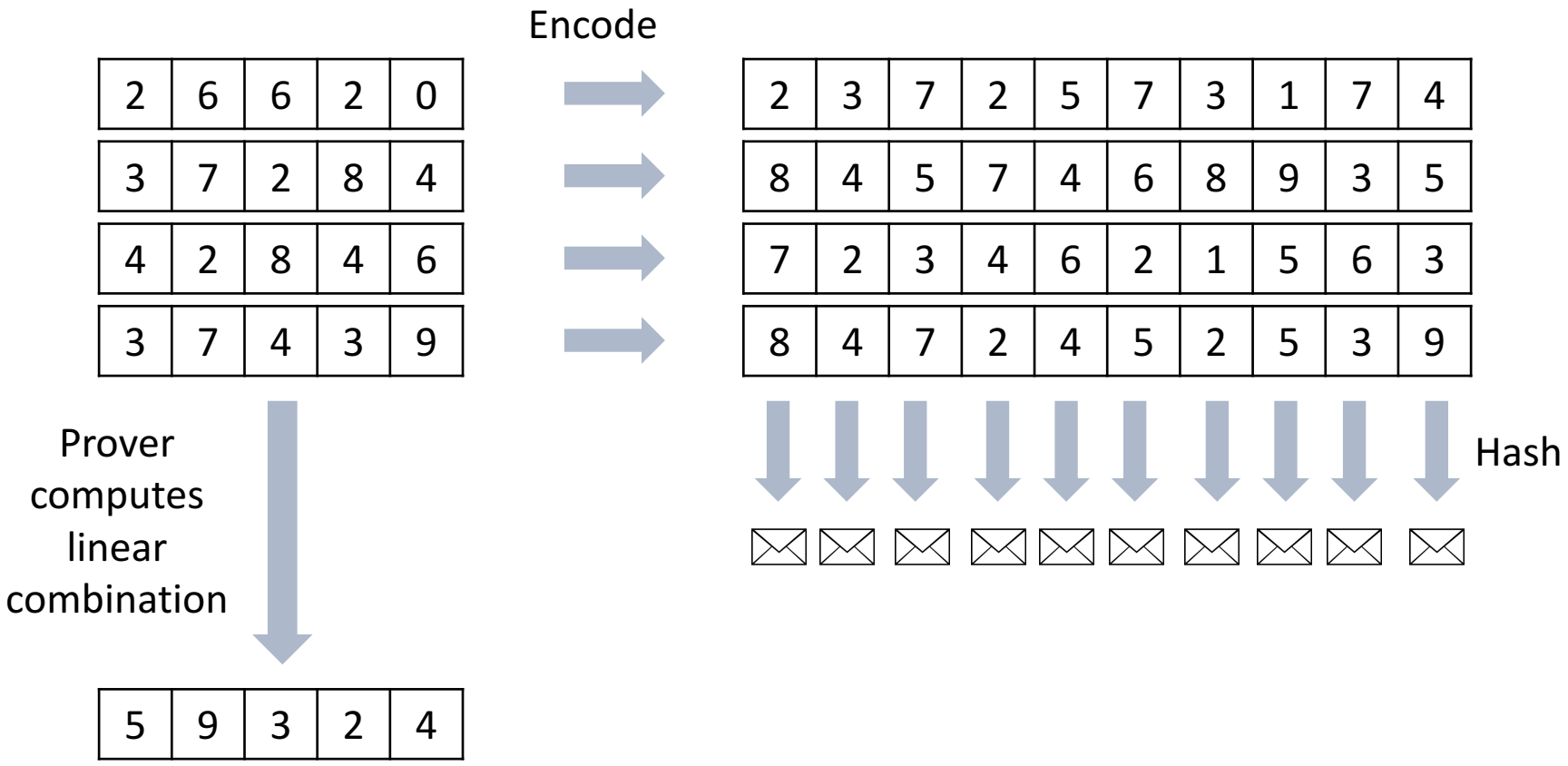
$z \leftarrow \mathsf{C}$

Compute linear combinations

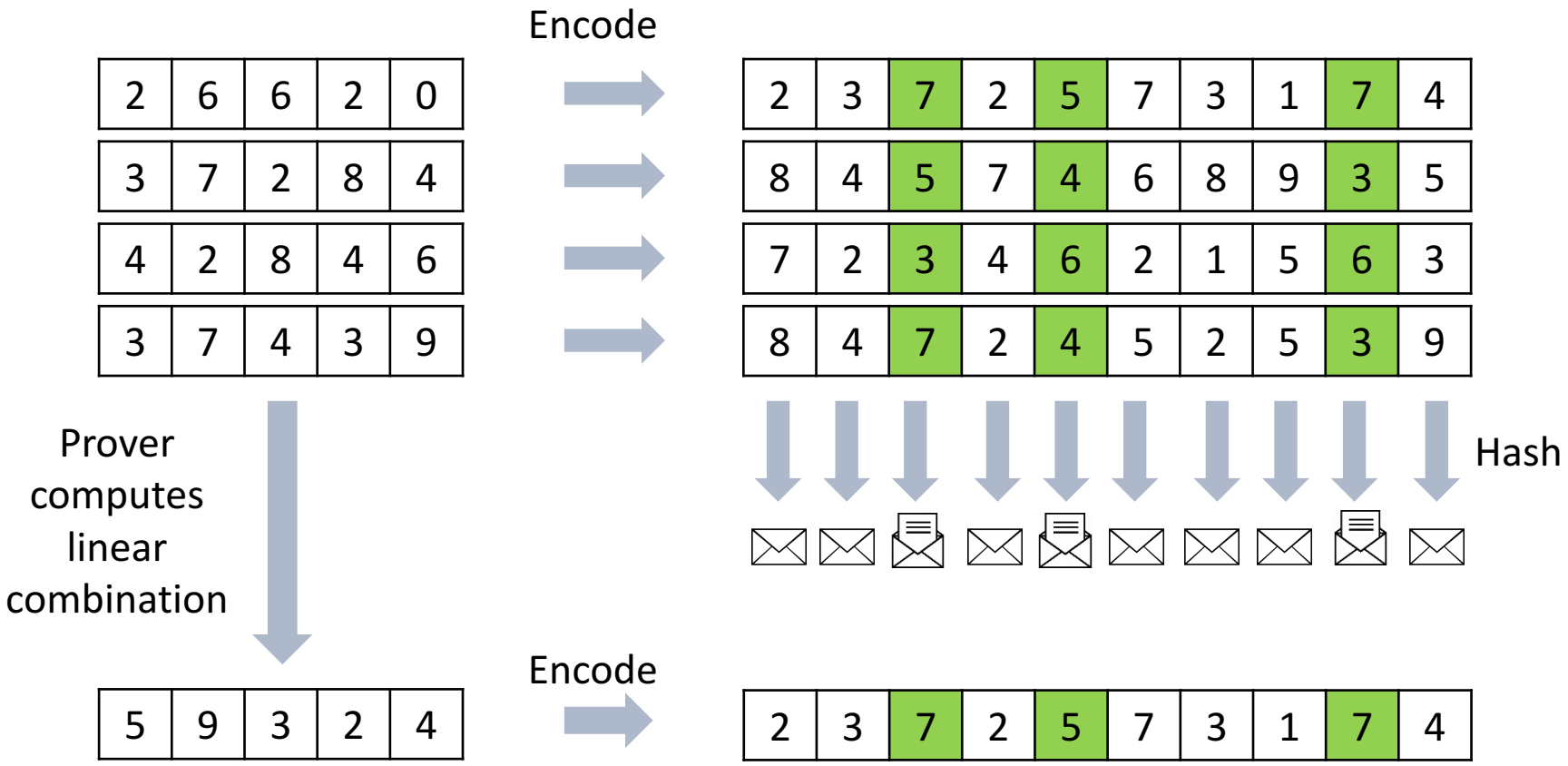
Linear combinations

Coefficients of linear combinations embed useful conditions

Committing


Encode
2 6 6 2 0
3 7 2 8 4
4 2 8 4 6
3 7 4 3 9
2 3 7 2 5 7 3 1 7 4
8 4 5 7 4 6 8 9 3 5
7 2 3 4 6 2 1 5 6 3
8 4 7 2 4 5 2 5 3 9
Hash
Prover computes linear combination
5 9 3 2 4

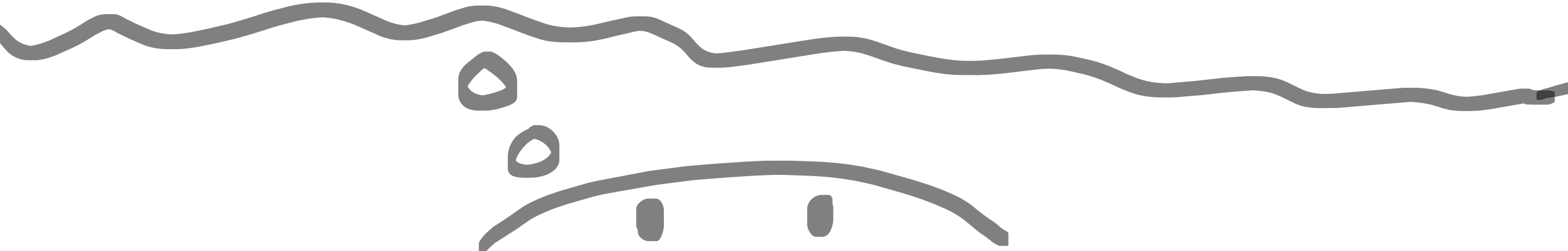
Checking Commitments



Verifier encodes and spot-checks columns
High minimum distance catches cheating

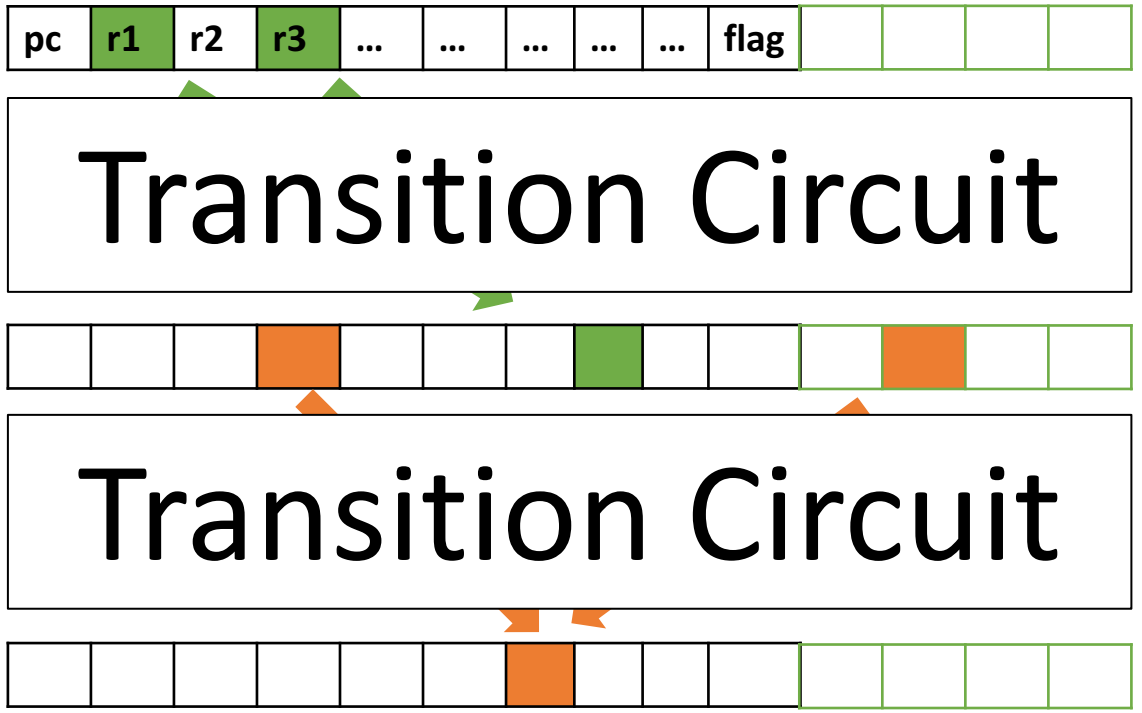
Linear-time Zero-knowledge Arguments (2017)

= Efficient Ideal Protocols + Linear-Time Encodable Error-Correcting Codes + Linear-Time Computable Hash Functions



Correct Instruction Execution

Check consistency of values across each time step



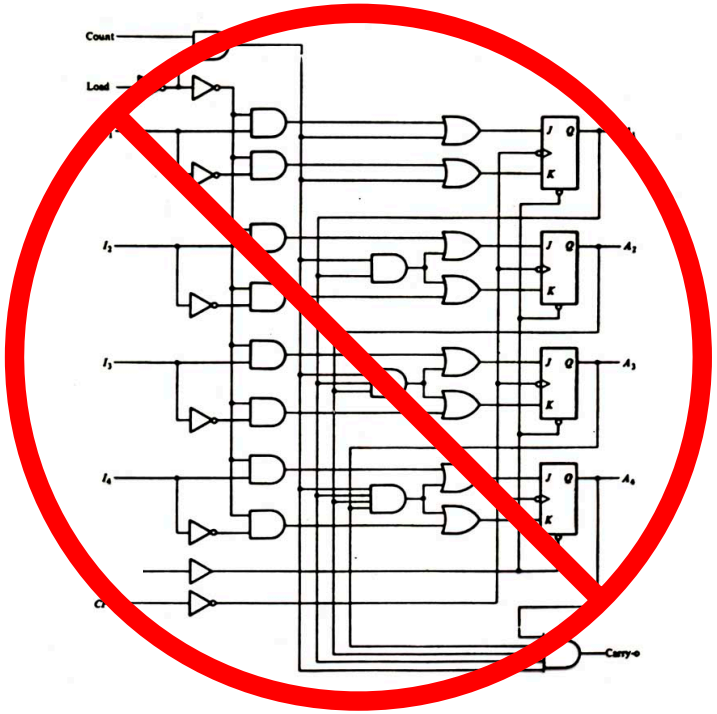
Covers all TinyRAM instructions

Constant size circuit

Give batch argument that each copy of circuit is satisfied

Word Decomposition

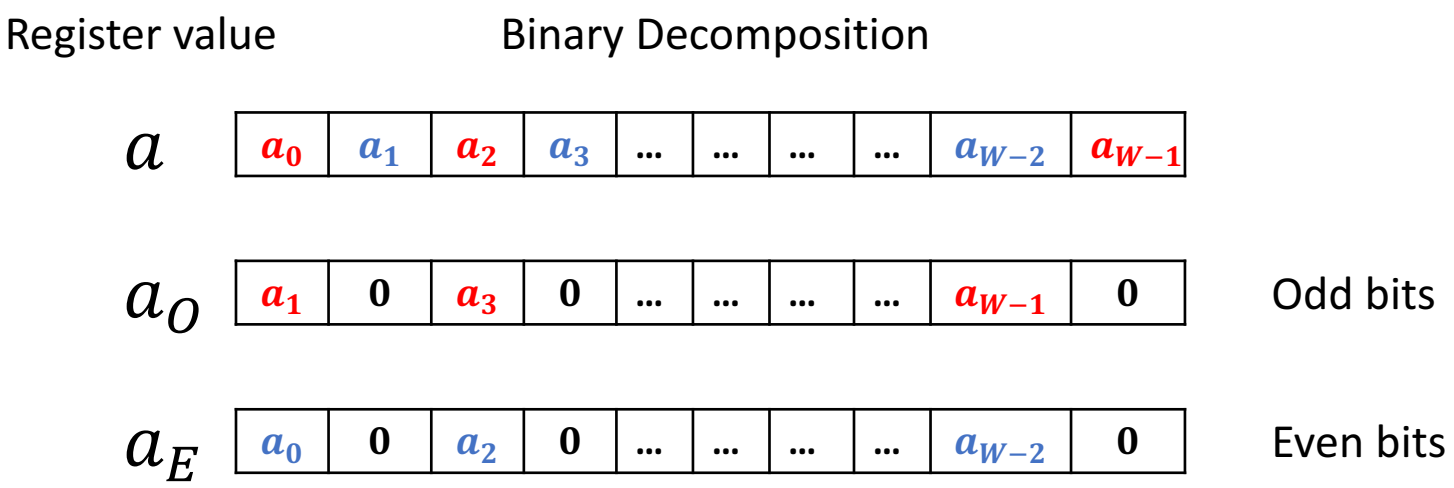
Avoid binary circuits when checking bitwise operations on non-binary field elements!



$$a, b \in \{0,1\}$$

$$a + b = 2(a \wedge b) + (a \oplus b)$$

Word Decomposition



$$a = 2a_O + a_E$$

Word Decomposition

a	a_0	a_1	a_2	a_3	…	…	…	…	a_{W-2}	a_{W-1}
a_O	a_1	0	a_3	0	…	…	…	…	a_{W-1}	0
a_E	a_0	0	a_2	0	…	…	…	…	a_{W-2}	0

$$a = 2a_O + a_E$$

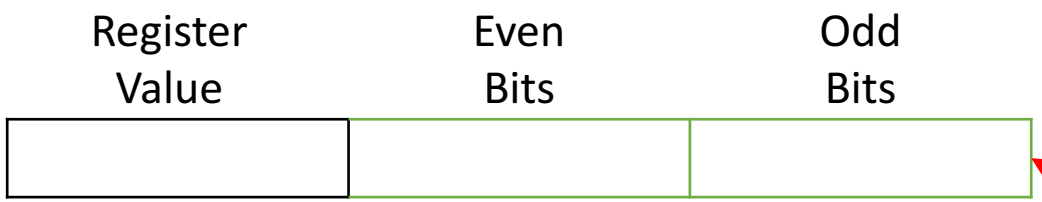
b	b_0	b_1	b_2	b_3	…	…	…	…	b_{W-2}	b_{W-1}
b_O	b_1	0	b_3	0	…	…	…	…	b_{W-1}	0
b_E	b_0	0	b_2	0	…	…	…	…	b_{W-2}	0

$$b = 2b_O + b_E$$

XORs in even bits	$a_0 \oplus b_0$	$a_0 \wedge b_0$	$a_2 \oplus b_2$	$a_2 \wedge b_2$	…	…	…	…	$a_{W-2} \oplus b_{W-2}$	$a_{W-2} \wedge b_{W-2}$	ANDs in odd bits

$$a_E + b_E$$

Look-up Argument



Decomposition Look-Up Table

Register Values	Even Bits	Odd Bits

All possible register values

Use zero-knowledge look-up argument to show all decompositions correct

Look-up Argument

Approach:

$b_1, b_2, \ldots, b_n$ already public

1. Commit to $a_1, a_2, \ldots, a_m, e_1, e_2, \ldots, e_n$

2. Prove in zero-knowledge that

Verify a 'square and multiply' algorithm in zero-knowledge

$$\prod_{i=1}^{m}(x - a_i) = \prod_{j=1}^{n}\left(x - b_j\right)^{e_j} \text{ for random } x$$

Memory Consistency

Approach:

$a_1, a_2, \ldots, a_m$ is a permutation of $b_1, b_2, \ldots, b_m$

$$\Leftrightarrow$$

$$\prod_{i=1}^{m}(X - a_i) = \prod_{i=1}^{m}(X - b_i)$$

Protocol similar to the look-up argument.

Summary

- Nearly-linear proving time
- Sublinear verification time
- New word decomposition technique for verifying binary operations over non-binary fields
- New look-up argument

Thanks!

Work	Prover Complexity	Verifier Complexity	Communication	Rounds	Assumption
BCTV14	$\Omega(T(\log T)^2)$	$\omega(L + \lvert v \rvert)$	$\omega(1)$	1	KoE
This Work	$O(\alpha T)$	$poly(\lambda)(\sqrt{T} + L + \lvert v \rvert)$	$poly(\lambda)(\sqrt{T} + L)$	$O(\log \log T)$	LT-CRHF

Security $2^{-\omega(\log \lambda)}$

Program Length L

Runtime Bound T

Public Input V