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Analysis of the Blockchain Protocol with Long Delays

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Nakamoto's blockchain

- Bitcoin introduced by Nakamoto in 2008
 - Decentralized payment system
 - Ledger maintained by the public in a decentralized manner
 - Attractive properties
 - Decentralization, Pseudonymity, Robustness ...

Top 10 Leaderboard

USD ▾

NAME	PRICE	24H CHG
 BTC	\$4,327.53	▲ 1.74%
 XRP	\$0.38	▼ -3.82%
 ETH	\$118.89	▼ -1.33%
 BCH	\$209.97	▼ -5.61%
 XLM	\$0.17	▲ 2.09%
 EOS	\$3.01	▼ -7.95%

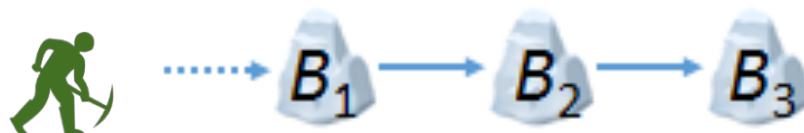
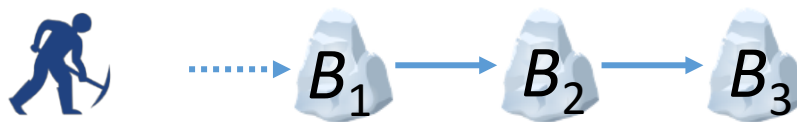




Nakamoto's blockchain

■ Blockchain

- Chain-structured ledger maintained by all the participants (miners)
 - Blocks can only be added to the end of the chain
- Basic security requirement
 - All the miners maintain the same record
 - Achieve **consensus** in the **permissionless** setting



permissionless

anyone can join (or leave)
the protocol execution



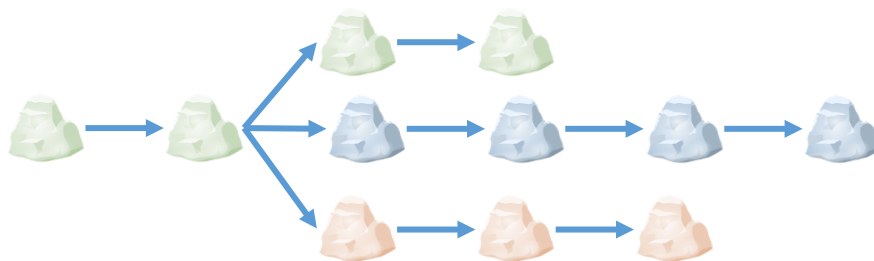
Nakamoto's blockchain

■ Proof of work (POW)

➤ Solve a “cryptographic puzzle”

- Integrity : More difficult for the adversary to modify the chain
- Synchronism : help the distributed miners to synchronize
 - Slowdown the generation of blocks
 - Longest chain rule

$$H(h||m||?) < D$$



Bitcoin Backbone Protocol [GKL15]

blockchain $C=(B_0, B_1, \dots, B_l)$

block $B_i = (h_{i-1}, m_i, r_i, h_i)$

$h_i = H(h_{i-1}||m_i||r_i)$, s. t. $h_i < D$



Nakamoto's blockchain



■ Security

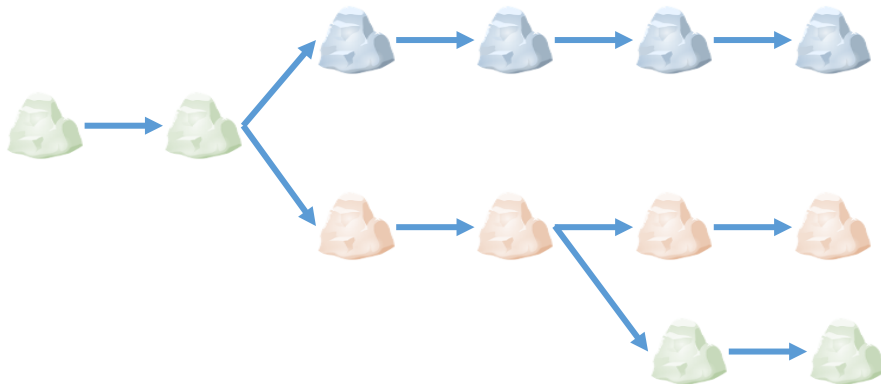
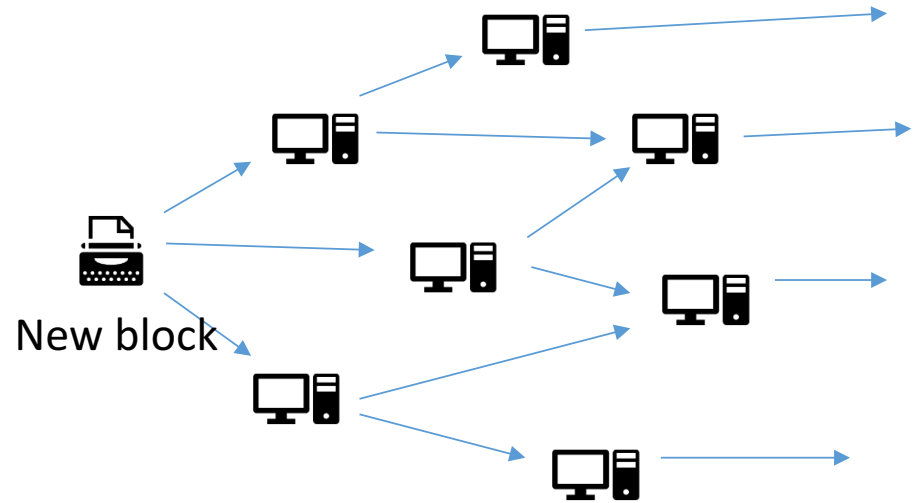
- Garay, Kiayias and Leonardos [GKL15] provide a rigorous analysis of blockchain protocol
 - Synchronous model
- Pass, Seeman and shelat [PSS17] analyze the security in an asynchronous network with a-priori bounded delay
 - Asynchronous model

Why consider the delay?



Blockchain protocol with delays

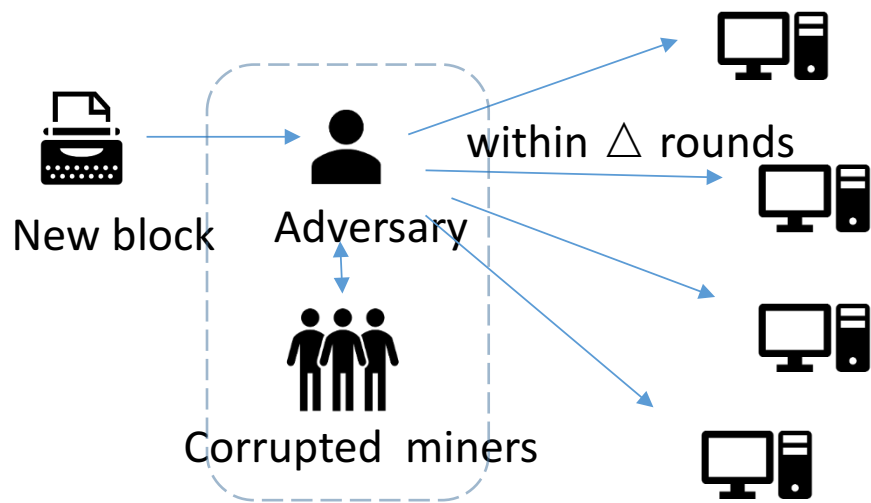
- Bitcoin P2P network
- Delays are inevitable



- The propagation delay in the network is the primary cause for blockchain forks [DW13]

Blockchain protocol with delays

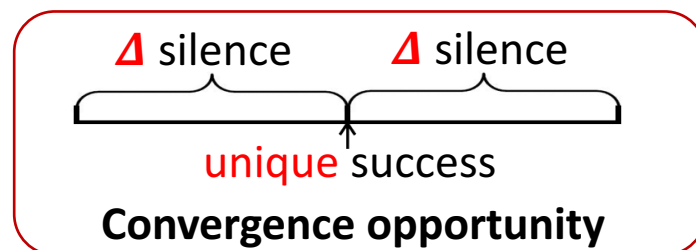
- Adversary in [PSS17]
- Responsible for the all message delivery
 - All the message can be delayed within Δ rounds
- Has certain factions of hash power

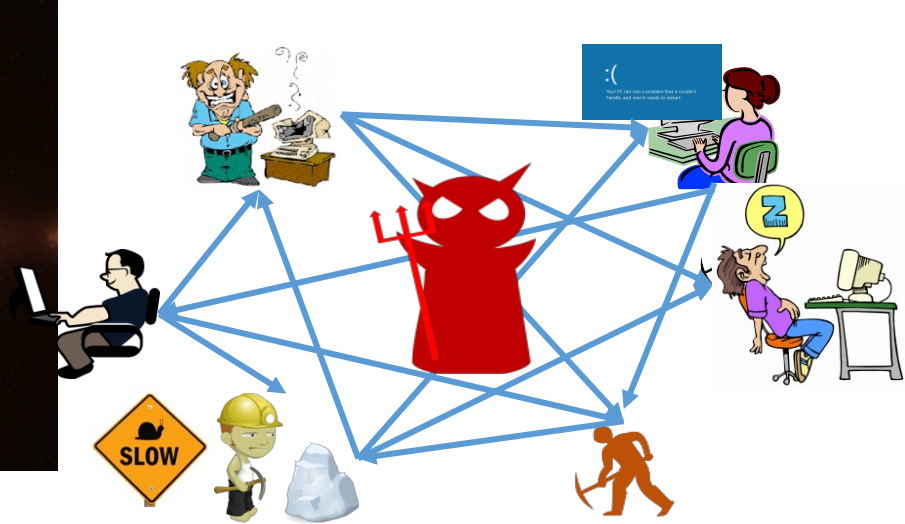


- Chain growth: $\frac{(1-\epsilon)f}{1+f\Delta}$, where $f \approx np$
- Consistency: T with probability $1 - \text{negl}(T)$
- Chain quality: $1 - (1 + \epsilon) \frac{tp(1+f\Delta)}{f}$

- **Limitation: $\Delta \ll O(1/np)$**
The proof holds for a relatively small delay only

n : the number of miners
 p : the probability that a miner succeeds in mining a block at a round





-
- A circular graph with 20 nodes arranged in a ring. The nodes are colored black or red. A node on the left is connected to several other nodes by blue arrows, representing a directed edge.

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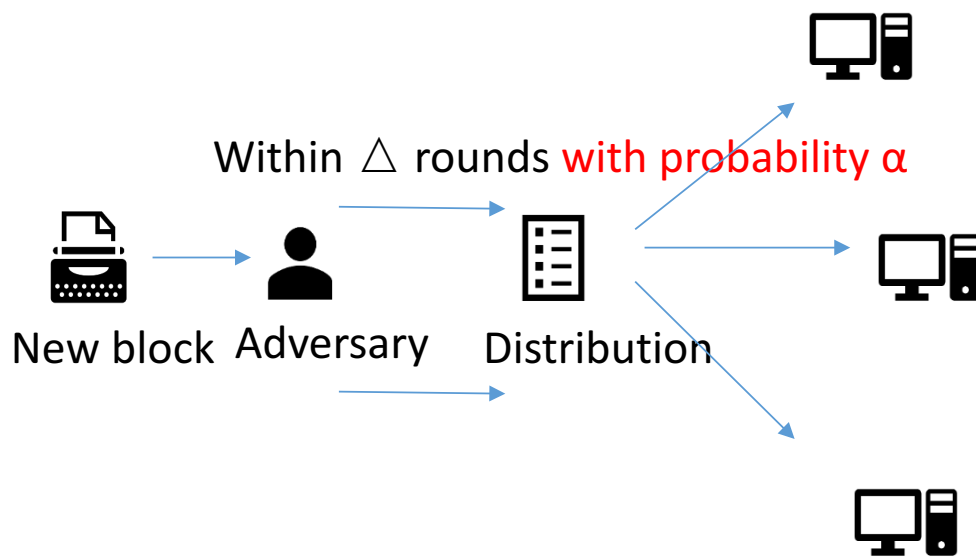


Blockchain protocol with delays

Is the blockchain protocol based on POW still secure in the asynchronous network, where long delay, say $\Delta \geq 1/np$, is allowed?

Our contribution

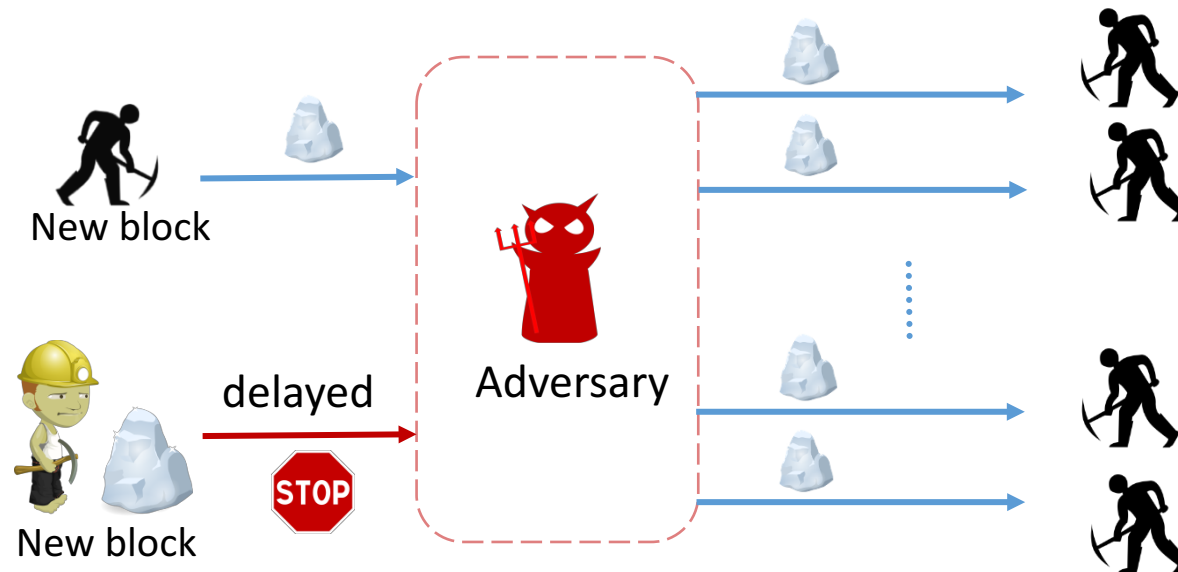
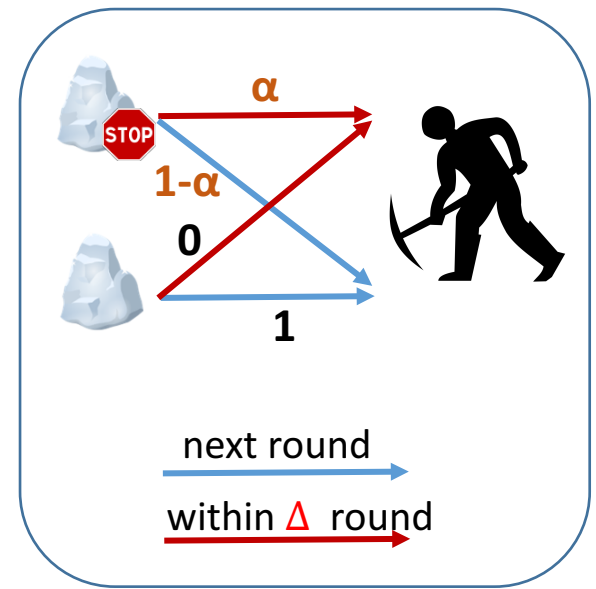
- Focus on the effect of long delay, especially $\Delta \geq 1/np$
 - Prove that the common prefix property and the chain growth property can still hold in our model when considering long delay
 - define chain growth and common prefix in a more subtle way
 - simplified proof method for POW based blockchain



Our blockchain model

■ The adversary A

- Deliver all messages sent by miners
- Delay the target chains with **probability α**
 - **Within Δ rounds**
- Do not have any hash power





Our blockchain model

- Modification to blockchain protocol
 - Consecutive blocks cannot be mined by the same miner (not the same mining pool)
 - a single miner
 - an independent communication node of the network
 - has a unit computational power
 - May lead to possible forks
 - In practice It is unlikely that a miner can mine two consecutive blocks
 - large number of miners n
 - small difficulty parameter p



Our blockchain model

Too weak?

■ Honest miners setting

- The adversary does not corrupt any miners (No hash power)
- Our model captures a class of practical attacks in the real world

■ For the adversary in a large-scaled blockchain protocol

- More difficult to control a sizable fraction of hashing power
- Much easier to disrupt communications among miners
- Present a concrete attack in which an adversary without any hash power may threaten the common prefix property

Security requirements



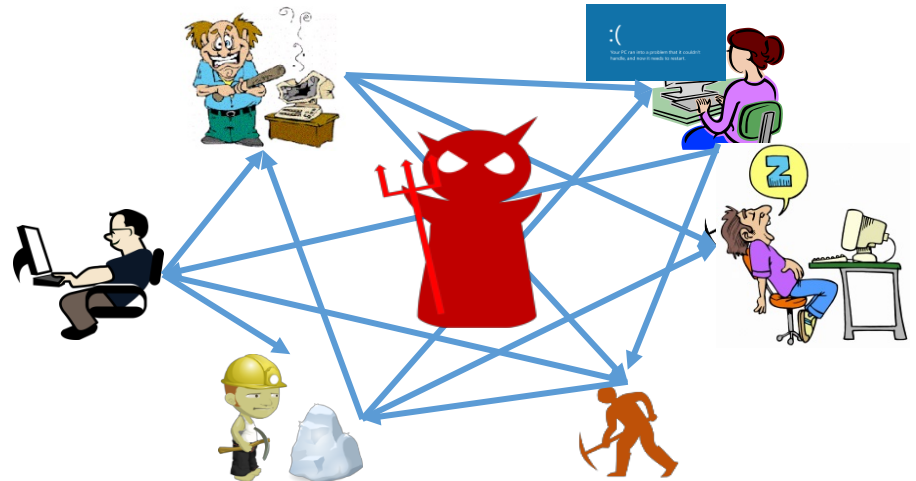
■ Chain Growth

- Previous work: the minimum length increase of **all honest miners'** chains during T rounds



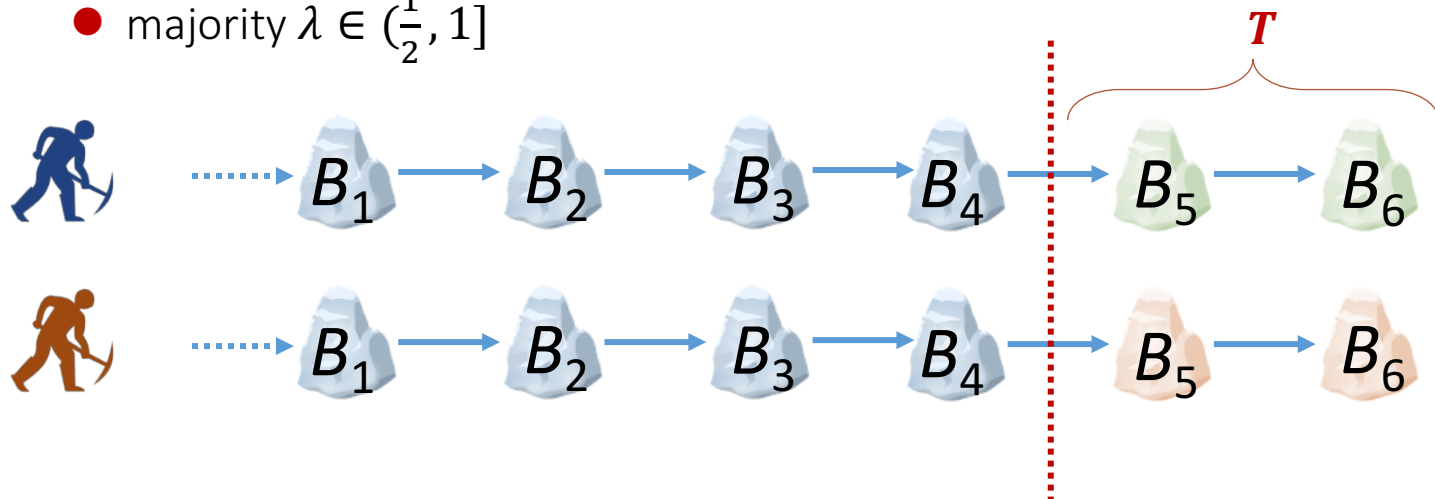
- Our work: the length increase of the **majority of honest miners'** chains
 - majority $\lambda \in (\frac{1}{2}, 1]$
 - Exclude the “bad” honest minority
 - Chain growth in [PSS17] is a special case of ours when $\lambda = 1$

Security requirements



Common Prefix

- Previous work: **All** the honest miners have the **same** history (prefix)
- Our work: **The majority** of the honest miners have the **same** history
 - Allow **some** miners' chains to be **inconsistent** with the main chain
 - majority $\lambda \in (\frac{1}{2}, 1]$



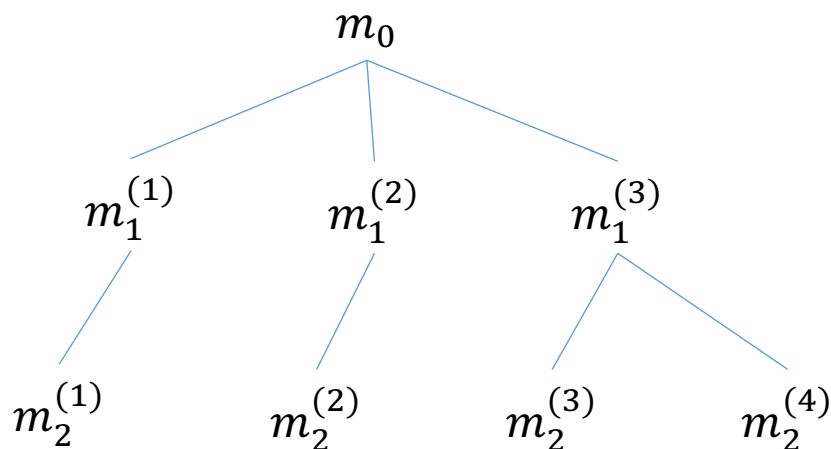


- How to capture the evolution of the main chains?



State of the Main Chain

- Tree_{MC} to capture the evolution of the main chains
 - Inspired by \mathbf{F}_{tree} model [PSS17], record all the branches (or forks)
 - Tree_{MC} in our model
 - Only store the current state of the main chains
 - Delayed chains are not recorded in Tree_{MC}
 - Basic operations: AddBlock, DeleteBlock



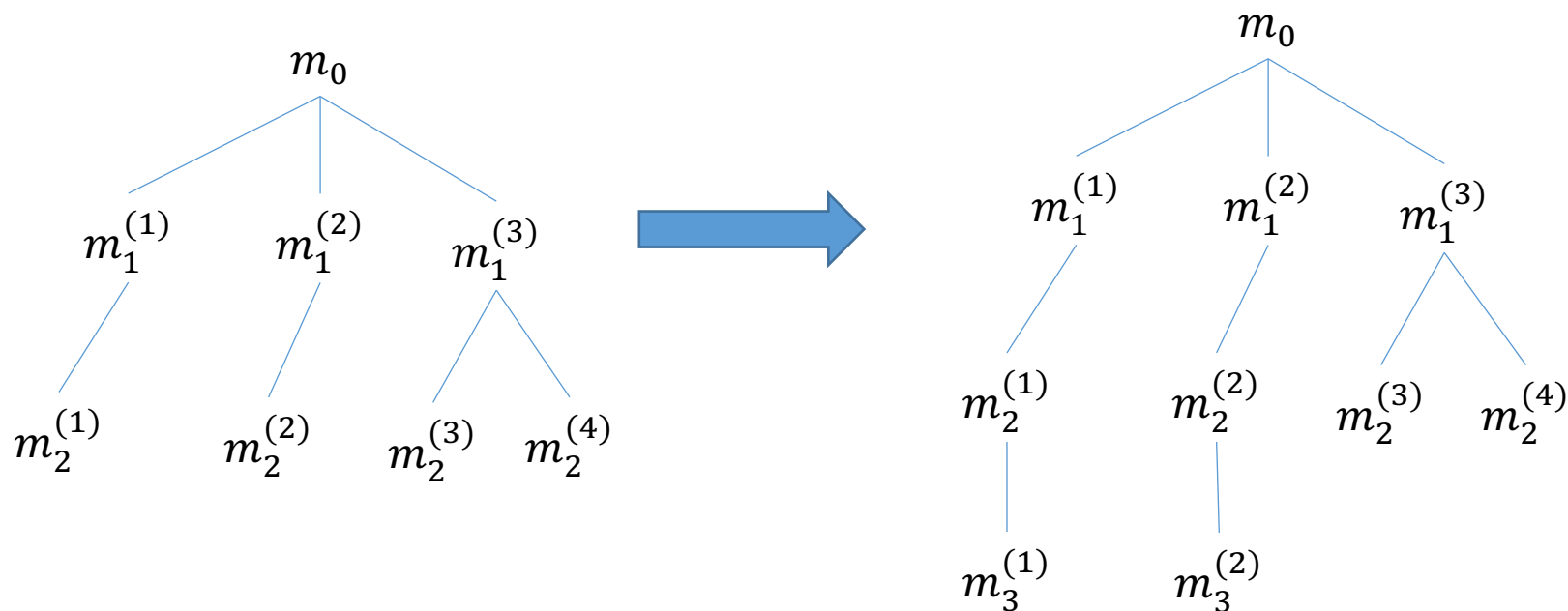
$$\begin{aligned} C_1 &= (m_0, m_1^{(1)}, m_1^{(1)}) \\ C_2 &= (m_0, m_1^{(2)}, m_2^{(2)}) \\ C_3 &= (m_0, m_1^{(3)}, m_2^{(3)}) \\ C_4 &= (m_0, m_1^{(3)}, m_2^{(4)}) \end{aligned}$$



State of the Main Chain

■ AddBlock:

- When the adversary broadcasts $C_1 = (m_0, m_1^{(1)}, m_2^{(1)}, m_3^{(1)})$ and $C_2 = (m_0, m_1^{(2)}, m_2^{(2)}, m_3^{(2)})$

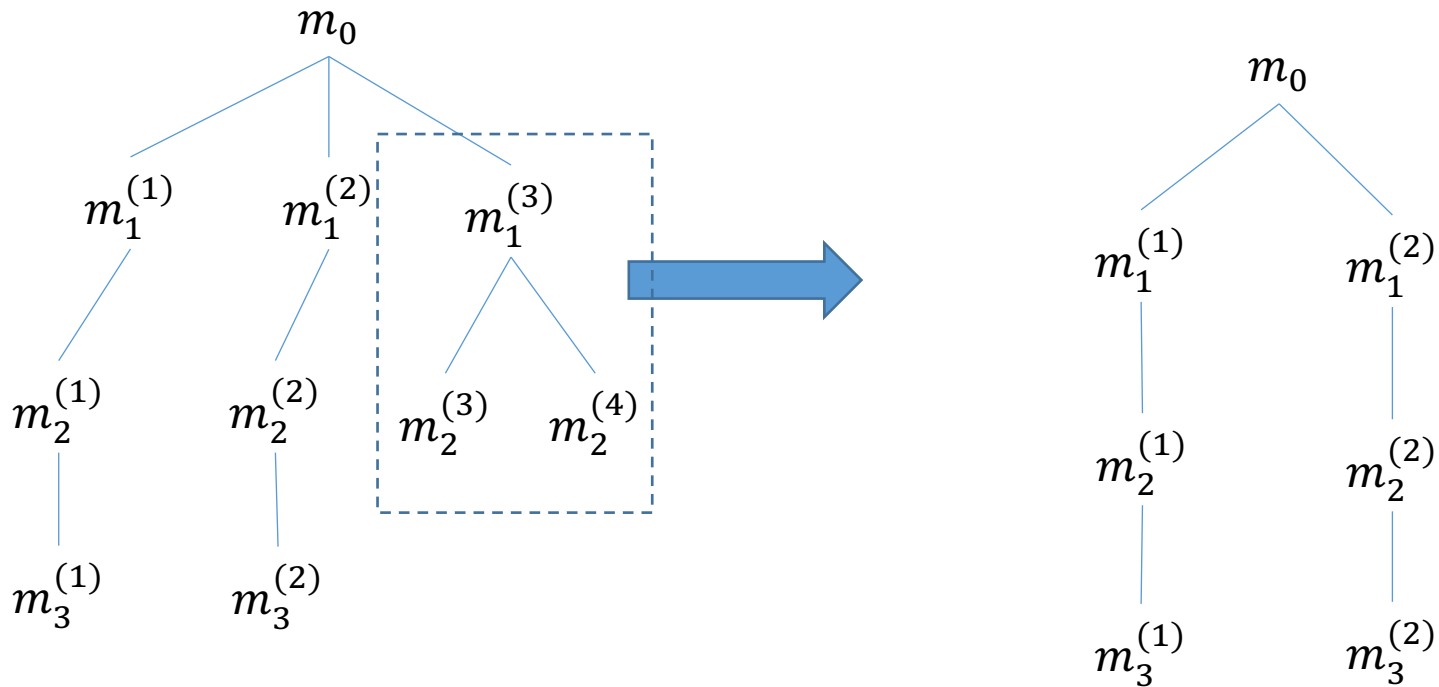




State of the Main Chain

■ DeleteBlock:

- Remove the useless nodes





Difference between Tree_{MC} and the miners' view

- Each miner has their own view of the main chain, which may be different with Tree_{MC}
- In terms of chain growth and common prefix, the difference is negligible
 - Reduced to the security of Tree_{MC}
 - Simple proof for Tree_{MC}
 - Useful properties on the depth of Tree_{MC}

Lemma 1. *Properties of Tree_{MC} .*

1. *If new blocks are successfully added to Tree_{MC} at the end of a round, then the depth of Tree_{MC} increases.*
2. *The depth of Tree_{MC} increases by at most 1 at each round.*
3. *If only one block is added to Tree_{MC} at the end of a round, then Tree_{MC} has only one branch and the depth increases by 1.*



Security proof

Chain Growth

Theorem 1 (*Chain growth*). Assume $1/2 < \lambda \leq 1 - 8\alpha p\Delta$. The blockchain protocol (Π, \mathcal{C}) has the chain growth rate $g = \frac{(1-\delta)f}{1+fE[R_{delay}^i]}$ with majority λ , where $f = 1 - (1-p)^n$, $E[R_{delay}^i] = \frac{\alpha - \alpha\omega^{\Delta-1}[\omega + \Delta(1-\omega^2)]}{1-\omega}$ and $\omega = 1 - (1-\alpha)f$.

Main idea of proof

$$g > \frac{t}{R_{mine} + R_{delay}}$$

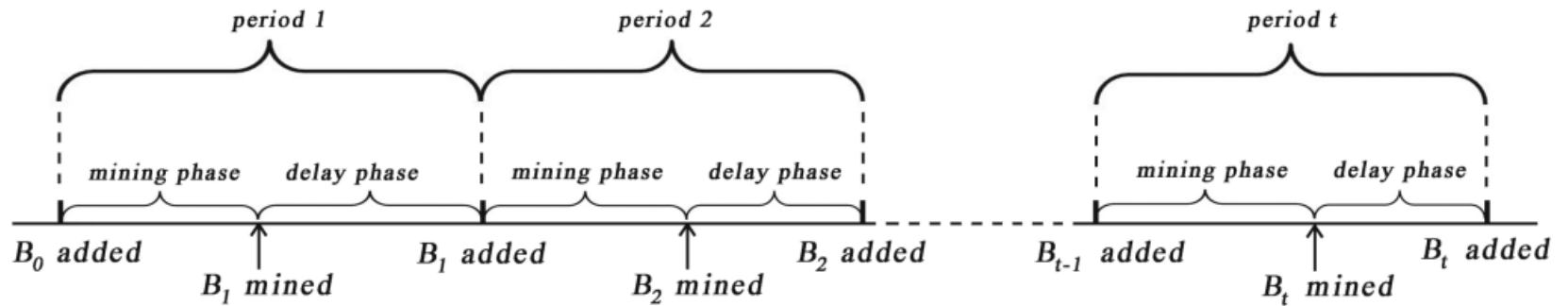


Fig. 1. The rounds during which t consecutive blocks are added to Tree_{MC}



Security proof

■ Common Prefix

Theorem 2 (*Common prefix*). Assume $0 < \alpha < 1 - np$ and $1/2 < \lambda \leq 1 - 8\alpha p\Delta$. The blockchain protocol (Π, \mathcal{C}) satisfies the common prefix property with parameter λ .

Main idea of proof

The event **converge**

- Only one miner succeeds in mining at round r^* .
- C^* is delayable while there is no new block mined in following Δ rounds
OR The chain C^* is undelayable

$$\Pr [\mathbf{converge}] > 1 - np(1 + \alpha\Delta)$$

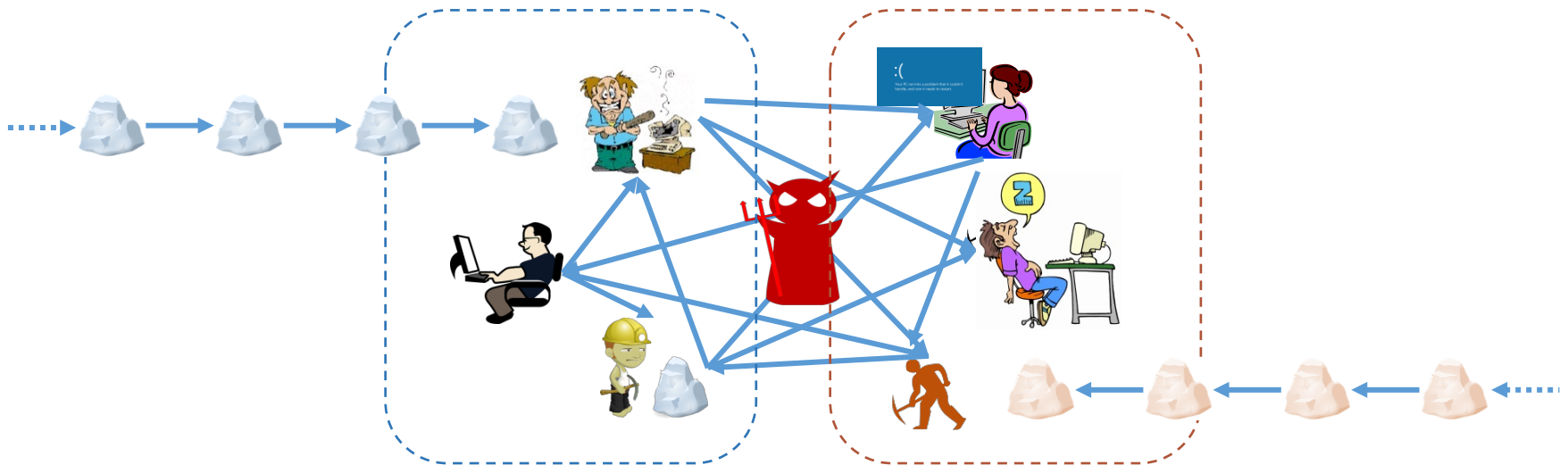
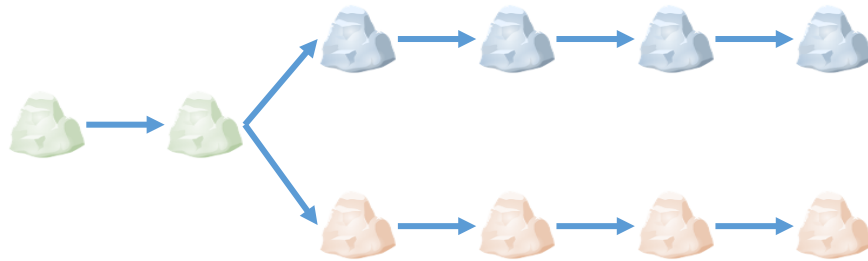
For Tree_{MC} with common prefix of depth $d-T$

$$1 - (np(1 + \alpha\Delta))^T$$



Long Delay Attack on Common Prefix

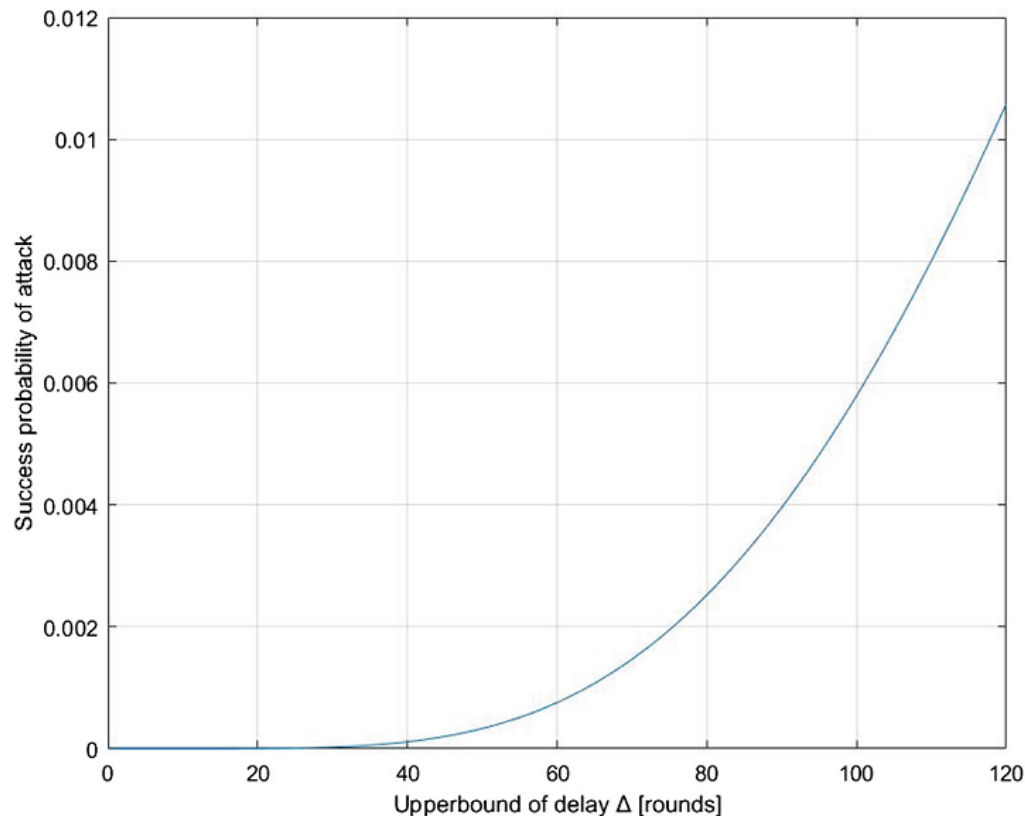
- Concrete attack on the common prefix of Tree_{MC}
 - when Δ and α are “too” large relative to a fixed T
 - Goal of attack: increase the length of the two branches by T





Long Delay Attack on Common Prefix

- With inappropriate parameters, adversaries without any hash power can threaten the common prefix property
 - For $\alpha = 0.8$ and $T = 6$, the success probability increases as Δ gets larger.



the success probability grows much faster when $\Delta > 60$ (10 min). When $\Delta > 120$ (20 min), the success probability can reach about 1%.



Future work

- Stronger security model
 - Convert honest miner setting to regular miner setting
- Robustness of blockchain for data storage
 - Provide reliable storage with provable robustness



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Thanks!
&
Questions?