On the Concrete Security of Goldreich's Pseudorandom Generator

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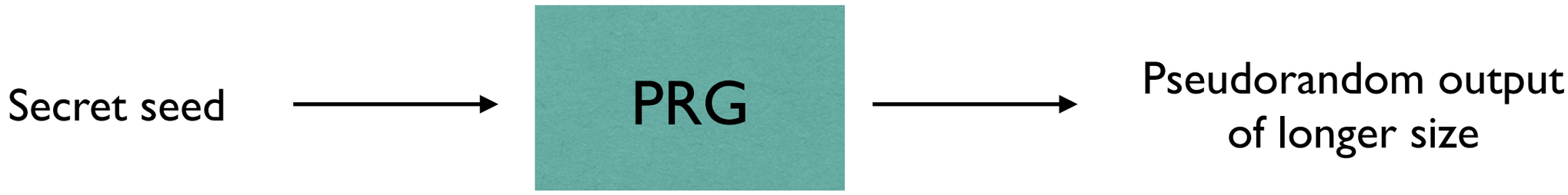
Goldreich Pseudorandom Generator (Goldreich TOCT 2000)



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$x_1, x_2, \ldots x_n \in \mathbb{F}_2$

Parameters (n, s)

Stretch $s > 1$

$m = n^s$

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Secret seed → **PRG** → Pseudorandom output of longer size

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x_1

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x_3

$\vdots$

x_n

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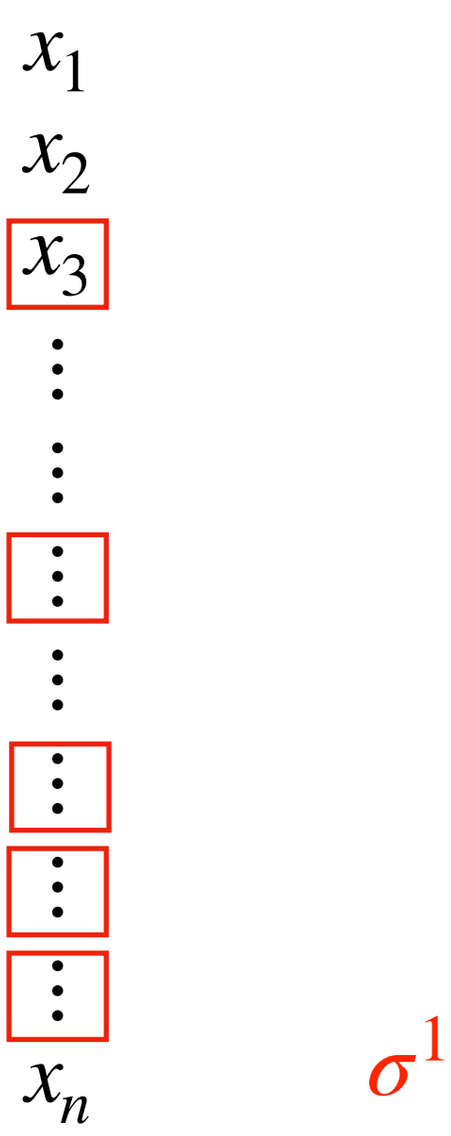

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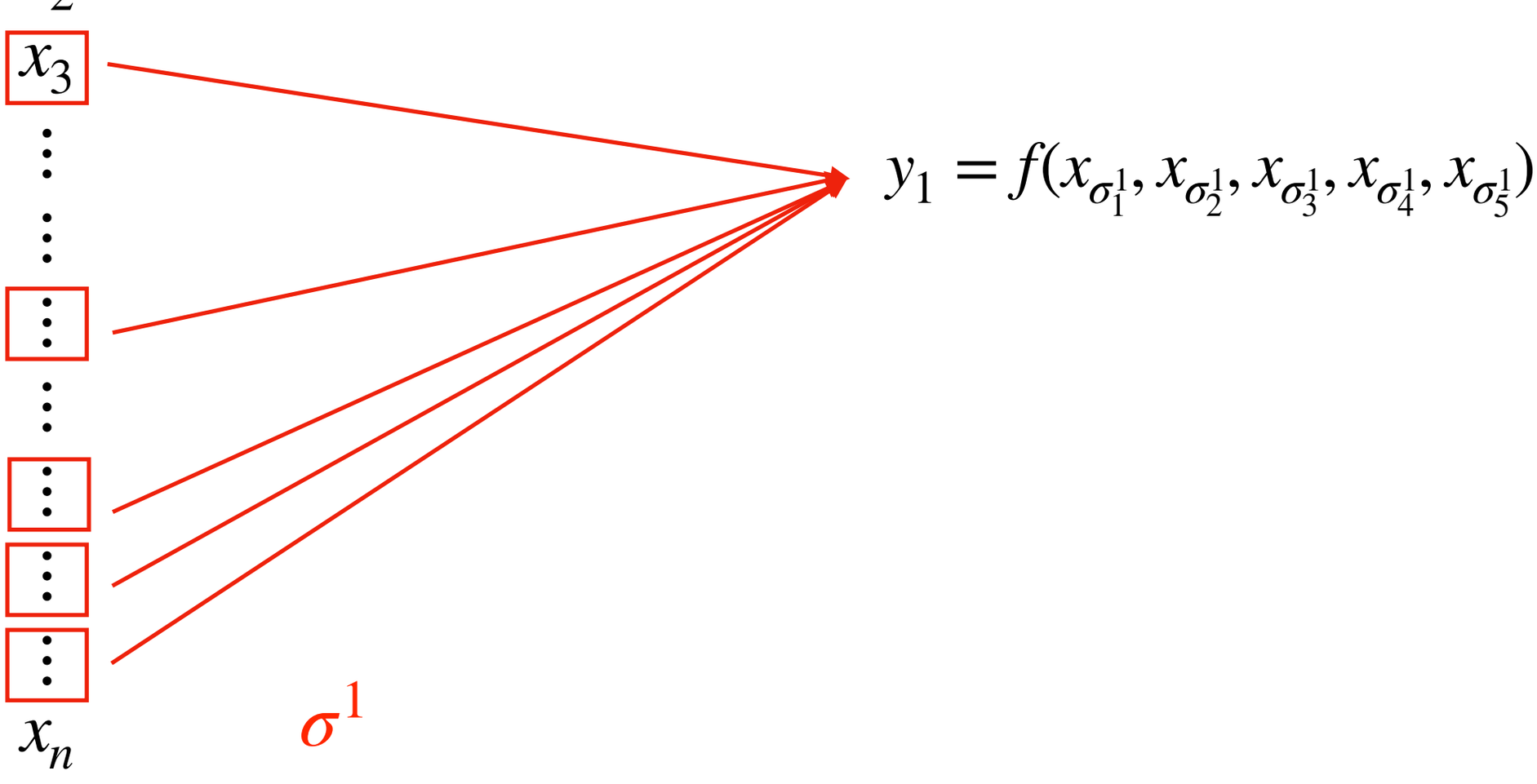

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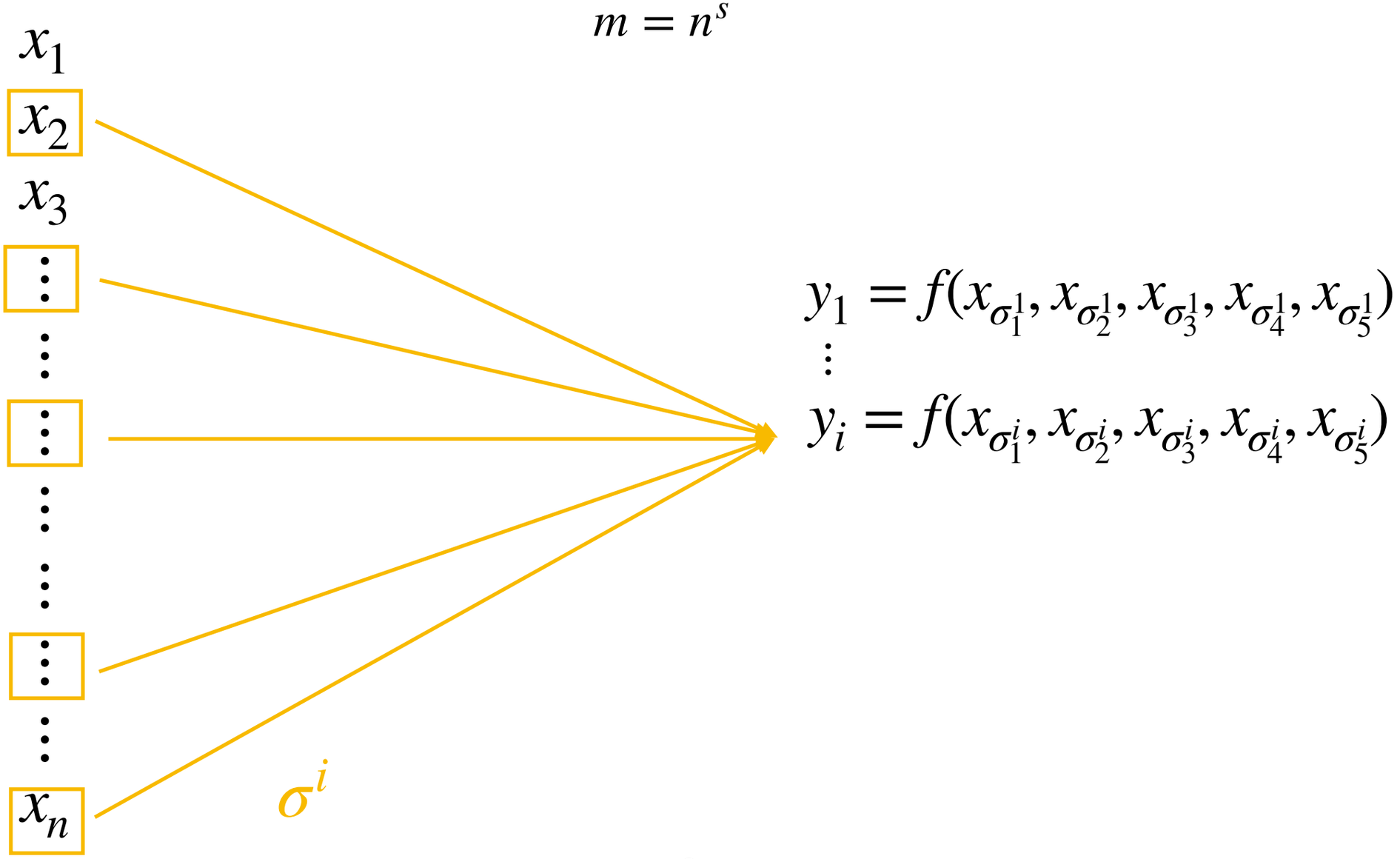

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$$y_1 = f(x_{\sigma_1^1}, x_{\sigma_2^1}, x_{\sigma_3^1}, x_{\sigma_4^1}, x_{\sigma_5^1})$$
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$$y_i = f(x_{\sigma_1^i}, x_{\sigma_2^i}, x_{\sigma_3^i}, x_{\sigma_4^i}, x_{\sigma_5^i})$$
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Public system

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Locality Cardinality of the subsets

- Here the locality is 5

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Public system

Predicate Function f

Boolean function of low degree

Goldreich Pseudorandom Generator

$x_1, x_2, \ldots x_n \in \mathbb{F}_2$

Secret seed

$$
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Public system

$y_1, y_2, \ldots y_m \in \mathbb{F}_2$

Public output

Goldreich Pseudorandom Generator

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Public system

$y_1, y_2, \ldots y_m \in \mathbb{F}_2$

Public output

Security properties Consider a uniformly random secret seed

1. **Pseudorandomness**

   $(y_1, y_2, \ldots y_m)$ is indistinguishable from uniform

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Security properties Consider a uniformly random secret seed

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2. **One wayness**

   Knowing the system and output, the probability to recover the seed is negligible

Predicate P5

$$f(x_{\sigma_1^i}, x_{\sigma_2^i}, x_{\sigma_3^i}, x_{\sigma_4^i}, x_{\sigma_5^i}) = x_{\sigma_1^i} + x_{\sigma_2^i} + x_{\sigma_3^i} + x_{\sigma_4^i} x_{\sigma_5^i}$$

Predicate P5

- Smallest locality 5
- Algebraic degree 2
- Algebraic immunity 2

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Mossel, Shpilka, Trevisan FOCS 2003

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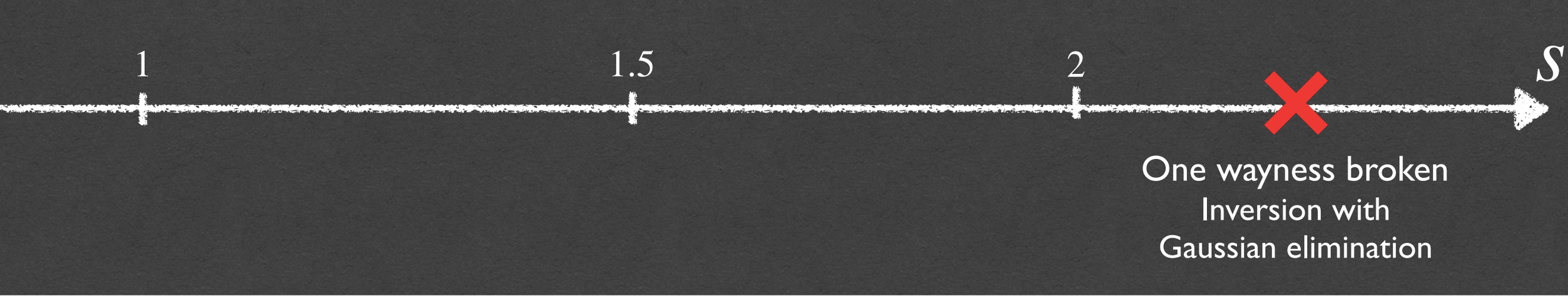

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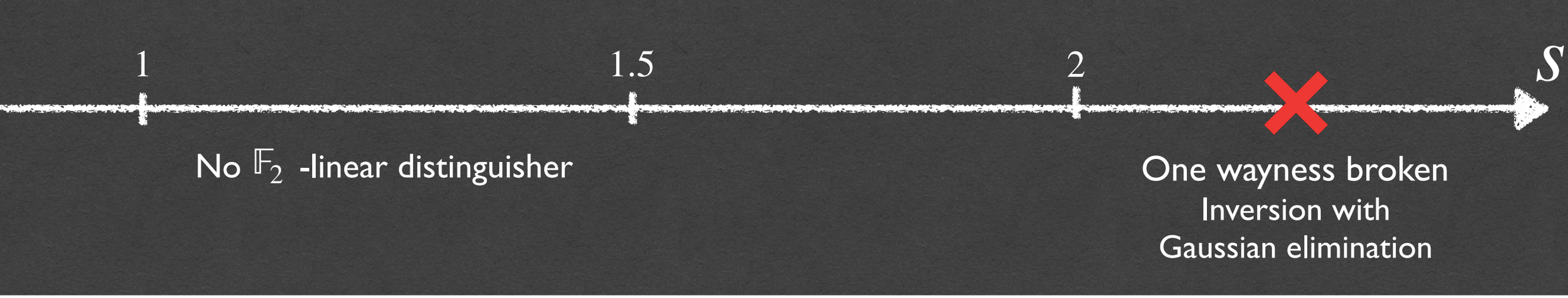

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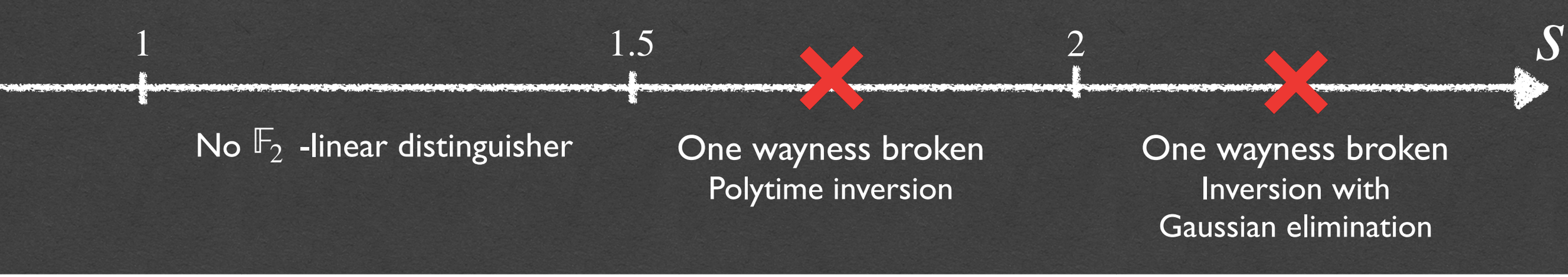

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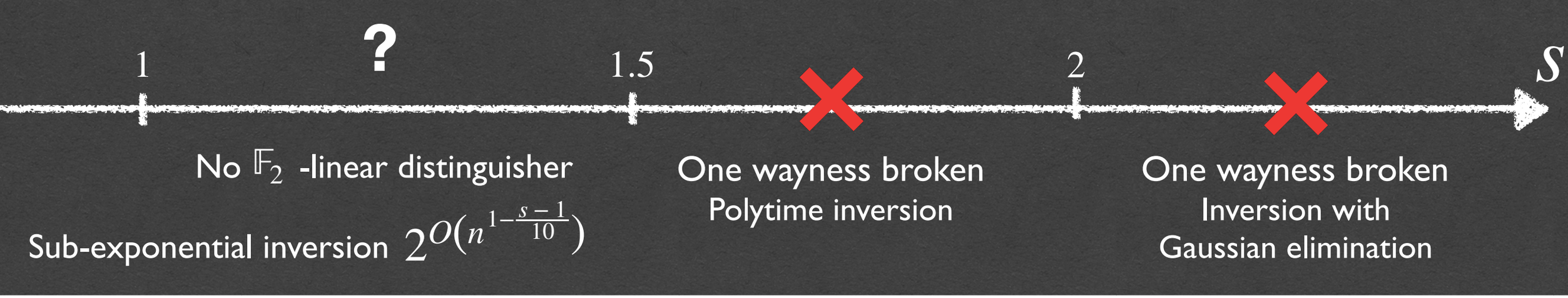

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Theoretical applications of Goldreich's PRG

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Cryptographic capsules

Boyle *et al.* ACM-CCS 2017

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New attacks

with a more fine-grained complexity estimation

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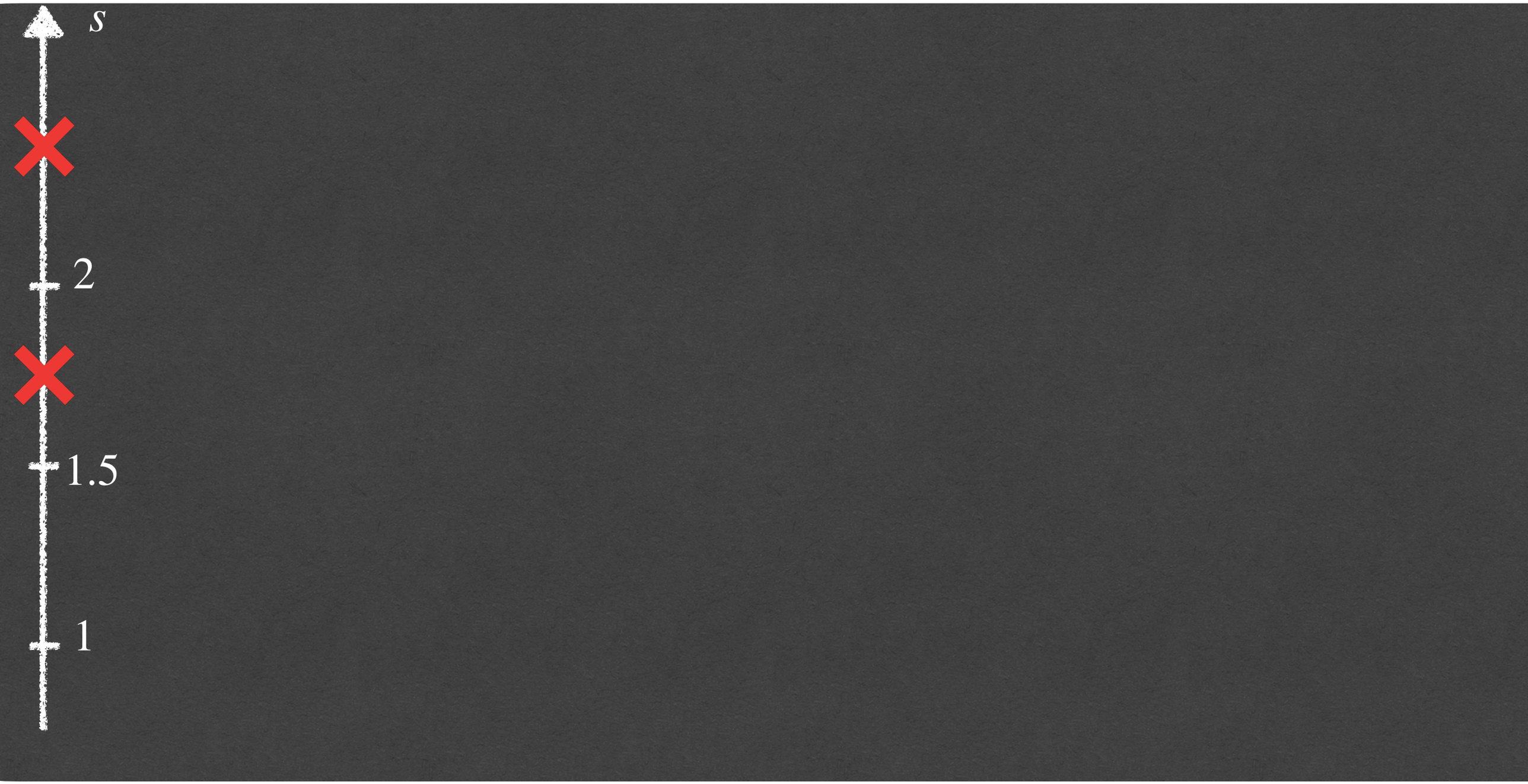

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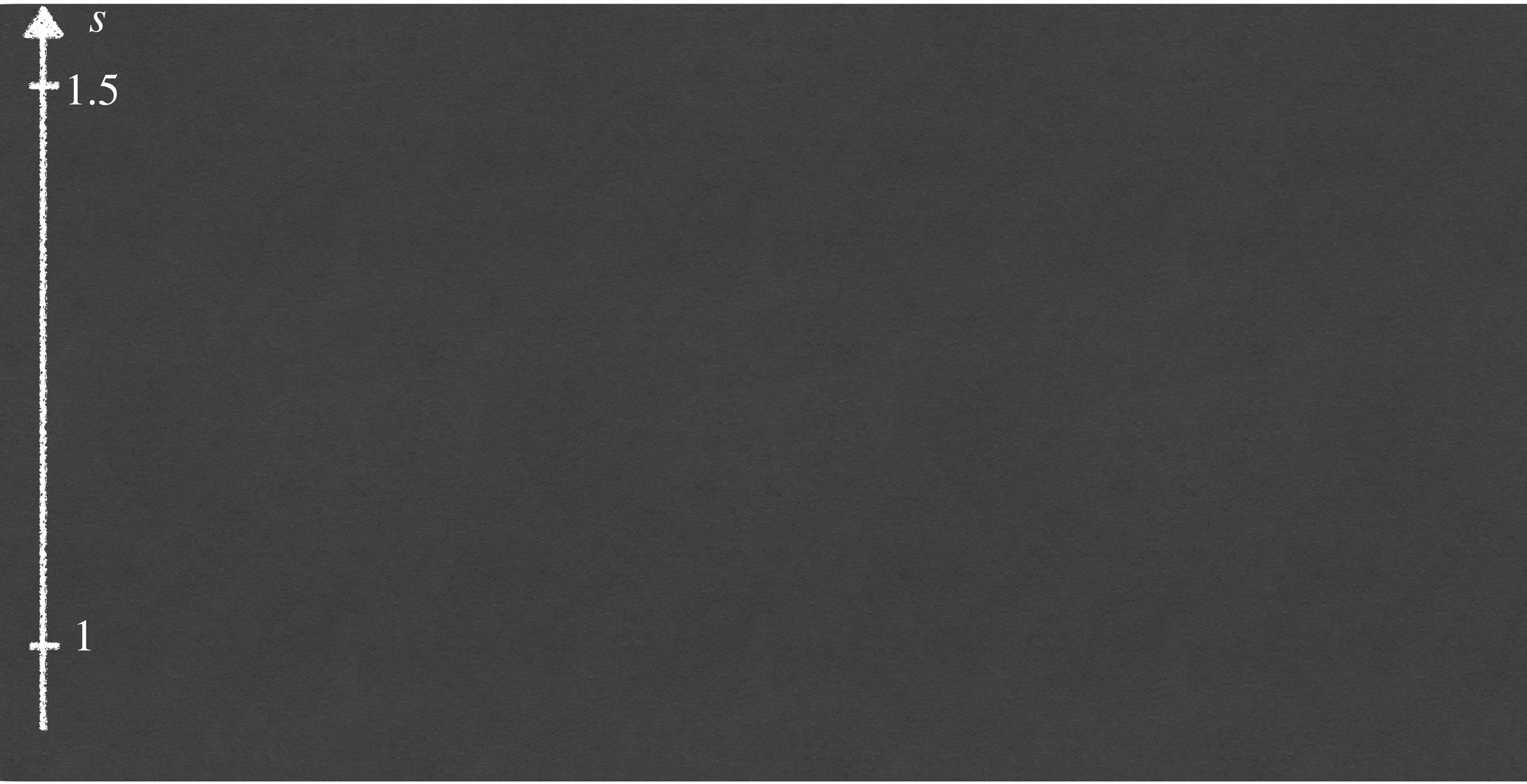

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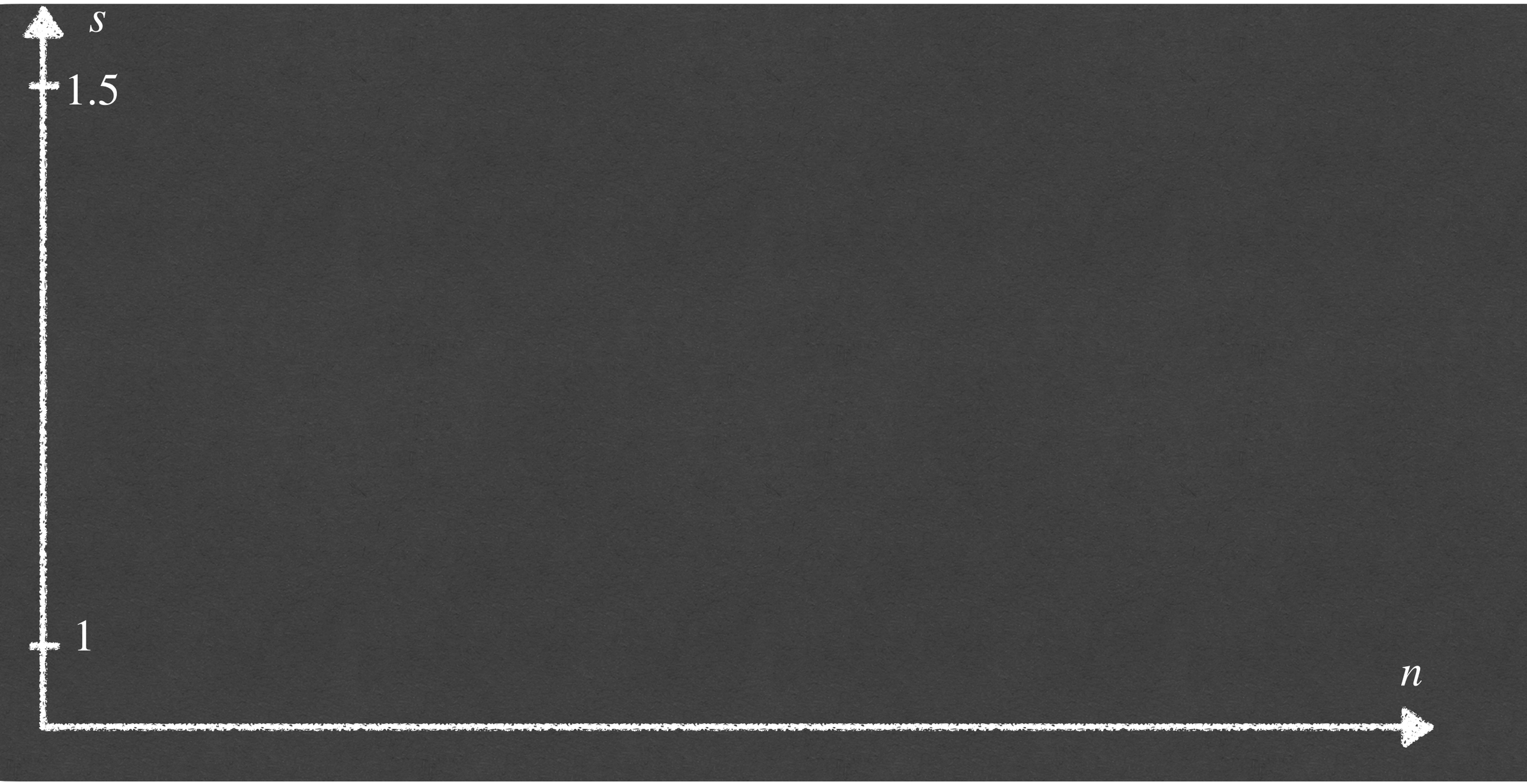

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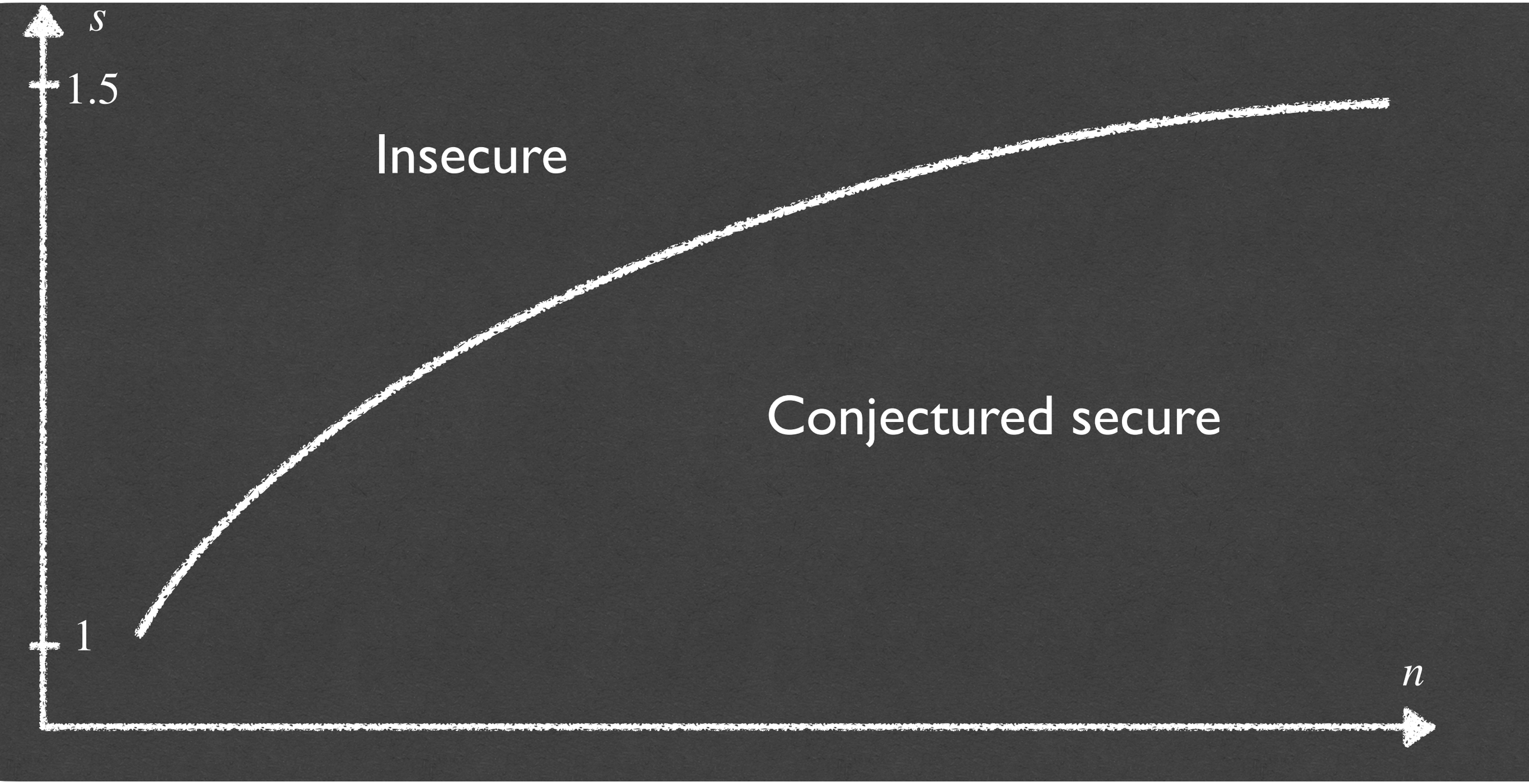

Outline

Guess-and-Determine attack

1. Collisions : linear equations for free

$$\begin{cases} x_3 + x_9 + x_2 + x_5x_1 = 1 \\ x_1 + x_8 + x_3 + x_{15}x_2 = 0 \\ x_4 + x_{10} + x_{12} + x_{14}x_6 = 0 \\ x_{17} + x_2 + x_1 + x_5x_1 = 1 \\ x_7 + x_1 + x_{11} + x_6x_5 = 1 \end{cases}$$

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Average number of collisions

$$\mathbb{E}(c) = m - \binom{n}{2} + \binom{n}{2}\left(\frac{\binom{n}{2} - 1}{\binom{n}{2}}\right)^m \in O(n^{2(s-1)})$$

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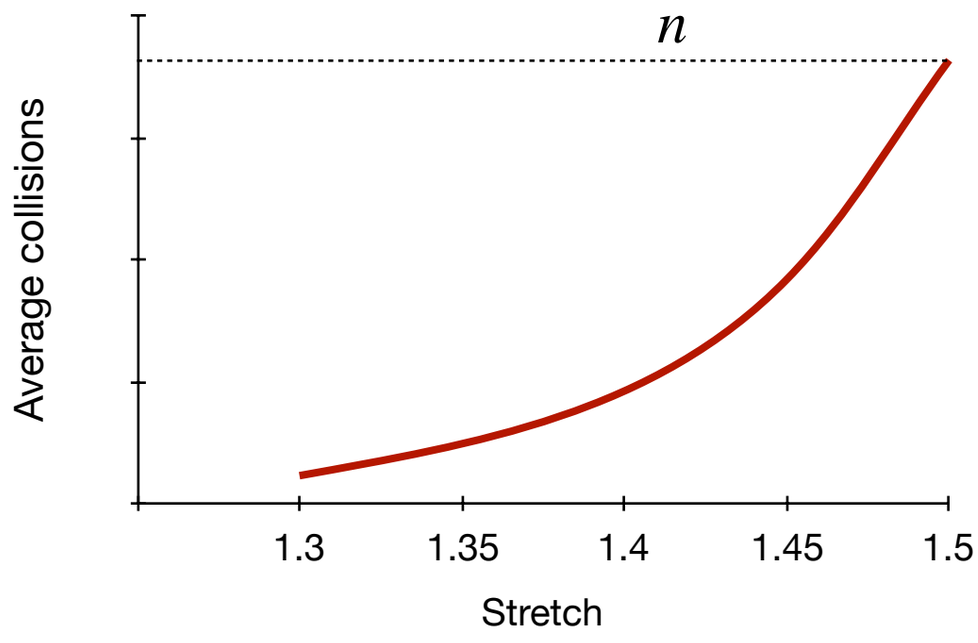

Guess-and-Determine attack

2 Complete the linear system obtained with guesses

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Guessing creates linear equations

Try $x_6 = 0$ and $x_6 = 1$

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Guessing creates linear equations

Try $x_6 = 0$ and $x_6 = 1$

Average number of necessary guesses

$$\left\lfloor \frac{n(n-c)}{2(m-c)+n} + 1 \right\rfloor \simeq O\left(\frac{n^{2-s}}{2}\right)$$

Guess-and-Determine attack

Idea introduced by Bettale PhD Thesis 2011

1. Derive all the collisions → Linear system Σ_1

Guess-and-Determine attack

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2. Compute a small subset $\mathcal{G}$ of guesses → Linear system Σ_2

   such that $|\Sigma_1| + |\Sigma_2| \gg n$

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3. Solve $\Sigma = \Sigma_1 \cup \Sigma_2$ for all elements in $\mathcal{G}$

   i.e. solve the system, find a candidate seed and check if it matches the public evaluation of the PRG.

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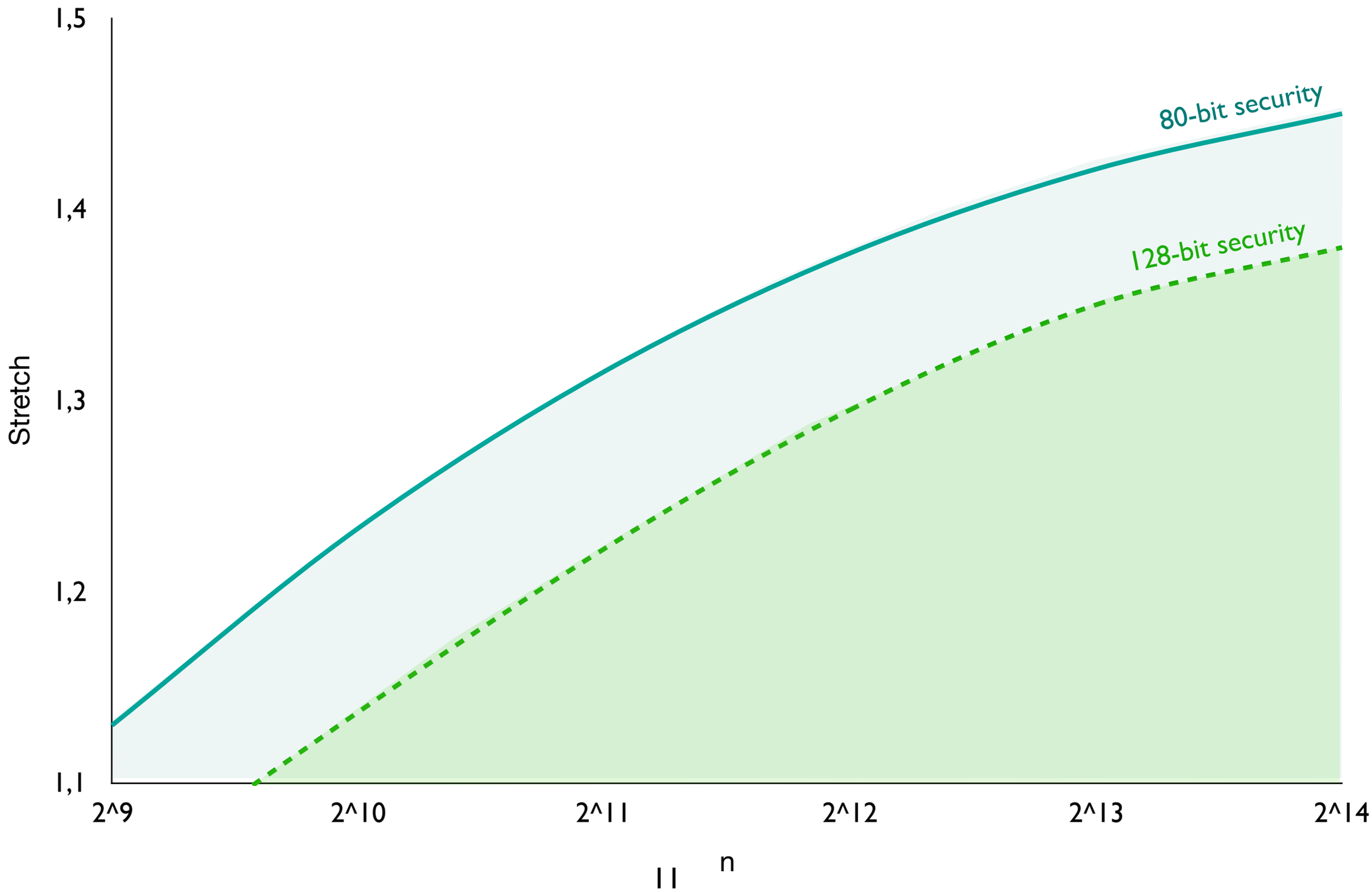
Total complexity $O\left(n^{\omega} 2^{\frac{n^2-s}{2}}\right)$

ω Exponent for solving linear systems

Guess-and-Determine attack



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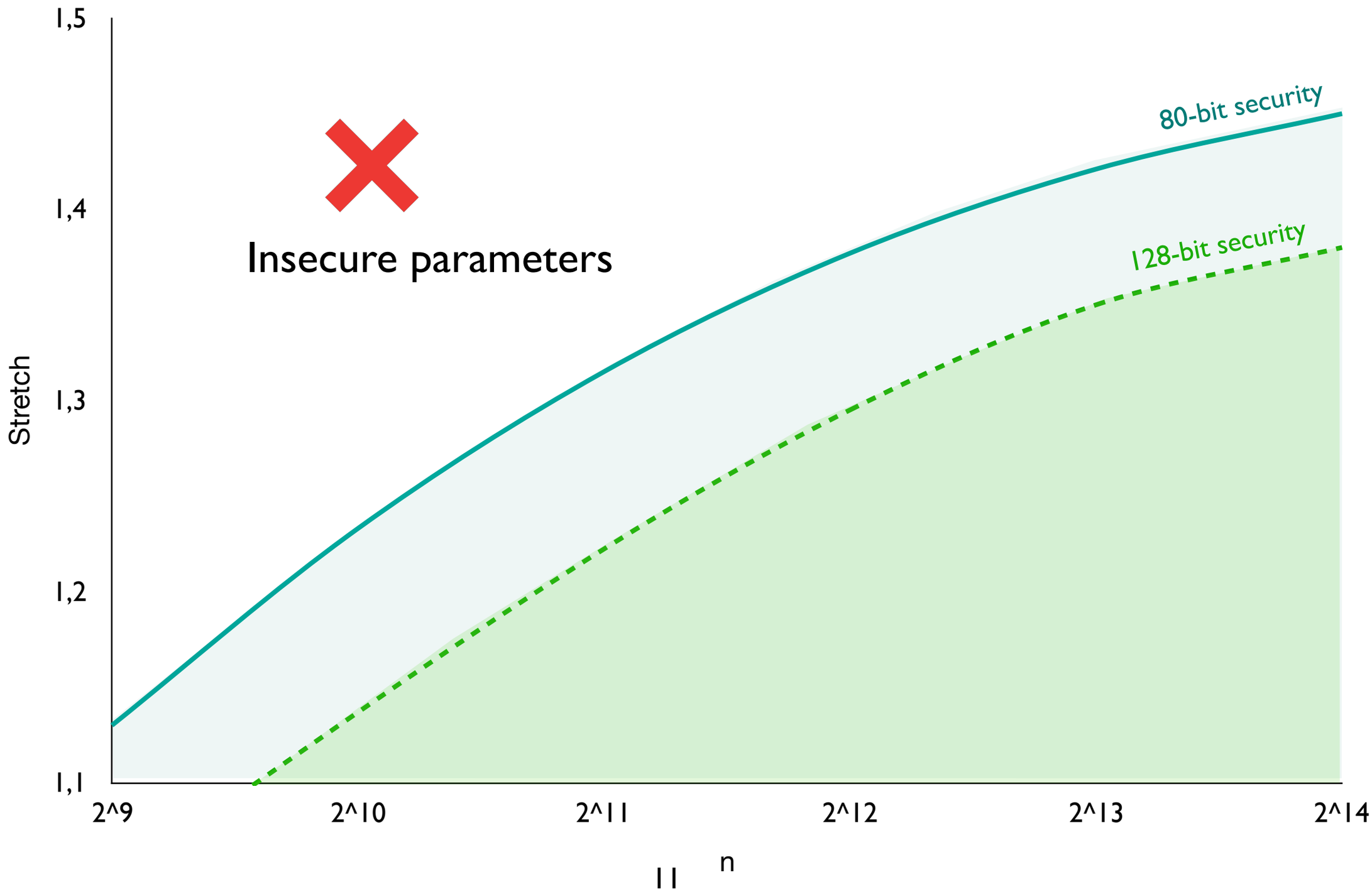

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Degree-two attack

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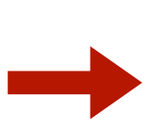
$$\vdots$$
$$y_i = x_{\sigma_1^i} + x_{\sigma_2^i} + x_{\sigma_3^i} + x_{\sigma_4^i} x_{\sigma_5^i}$$
$$\vdots$$
$$y_m = x_{\sigma_1^m} + x_{\sigma_2^m} + x_{\sigma_3^m} + x_{\sigma_4^m} x_{\sigma_5^m}$$

Public system



$$y_1 = x_{\sigma_1^1} + x_{\sigma_2^1} + x_{\sigma_3^1} + x_{\sigma_4^1} x_{\sigma_5^1}$$
$$\vdots$$
$$y_i = x_{\sigma_1^i} + x_{\sigma_2^i} + x_{\sigma_3^i} + x_{\sigma_4^i} x_{\sigma_5^i}$$
$$\vdots$$
$$y_m = x_{\sigma_1^m} + x_{\sigma_2^m} + x_{\sigma_3^m} + x_{\sigma_4^m} x_{\sigma_5^m}$$

Create many additional quadratic equations in order to be able to linearize

$$X_{i,j} \leftarrow x_i x_j$$

Degree-two attack

1. Create degree 2 equations

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$$\begin{cases} x_3 + x_9 + x_2 + x_5x_1 = 1 \\ x_1 + x_8 + x_3 + x_{15}x_2 = 0 \\ x_4 + x_{10} + x_{12} + x_{14}x_6 = 0 \\ x_{17} + x_2 + x_1 + x_5x_1 = 1 \\ x_7 + x_1 + x_{11} + x_6x_5 = 1 \end{cases}$$

$\times x_5$ → $x_5(x_3 + x_9 + x_2 + x_1) = x_5$

$\times x_1$ → $x_1(x_3 + x_9 + x_2 + x_5) = x_1$

Each equation can create 2 more equations

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Semi collision S-polynomial

$\times x_6$, $\times x_1$:

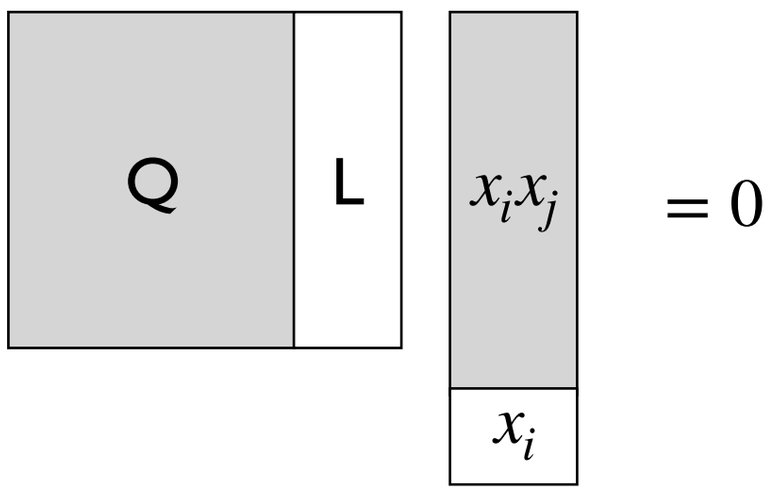
$$x_6(x_{17} + x_2 + x_1 + x_5x_1) + x_1(x_7 + x_1 + x_{11} + x_6x_5) = x_6 + x_1$$

$$x_6(x_{17} + x_2 + x_1) + x_1(x_7 + x_1 + x_{11}) = x_6 + x_1$$

Degree-two equation

Degree-two attack

2 Try to solve $X_{i,j} \leftarrow x_i x_j$



→ Q and L are very sparse

Degree-two attack

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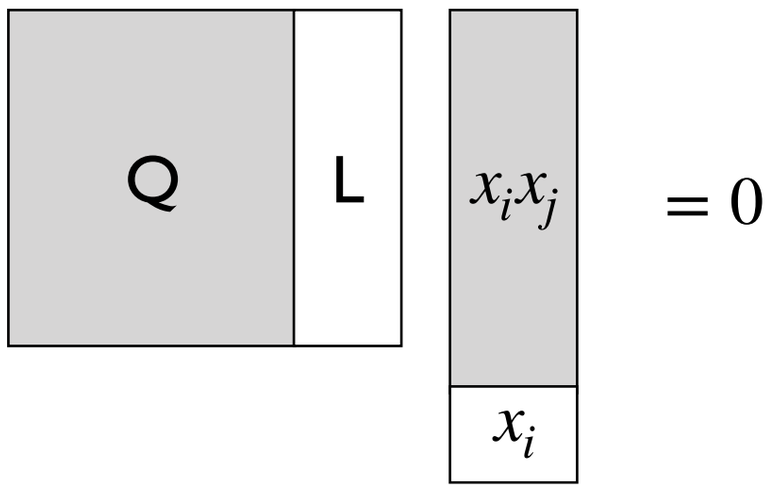

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When $\mathcal{N}_{eq}(n, s) \approx \mathcal{N}_{var}(n)$, Q|L is full rank and the secret seed can be recovered

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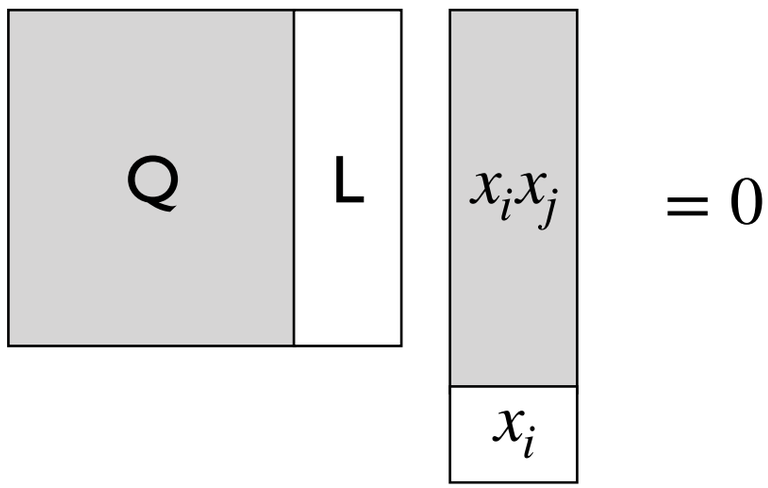

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When $\mathcal{N}_{eq}(n, s) \approx \mathcal{N}_{var}(n)$, Q|L is full rank and the secret seed can be recovered

Using heuristic assumptions (counting equations, linear independence), we were able to define a function f^* such that

$s \geq f(n) \implies$ the degree 2 attack succeeds with high probability

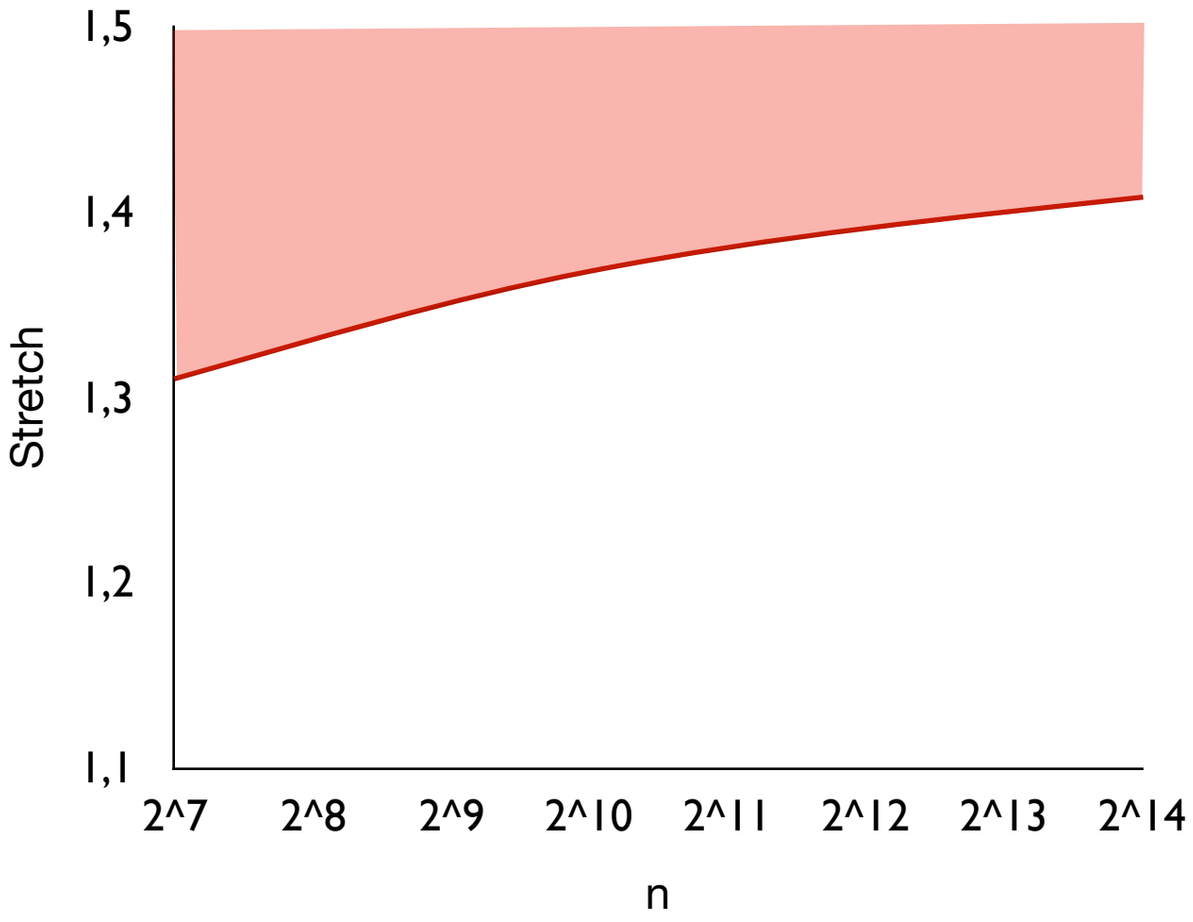
Conjectured degree 2 linearization (experimentally checked for small n)

*We thank Henri Gilbert and Guénaël Renault for helpful insights with this approach

Gröbner basis approach

Conjectured polynomial attack

For $s \geq f(n)$, the degree of regularity of the Gröbner basis computation is 3 with a degree 2 final resolution



➡ complexity $O(n^{2\omega})$

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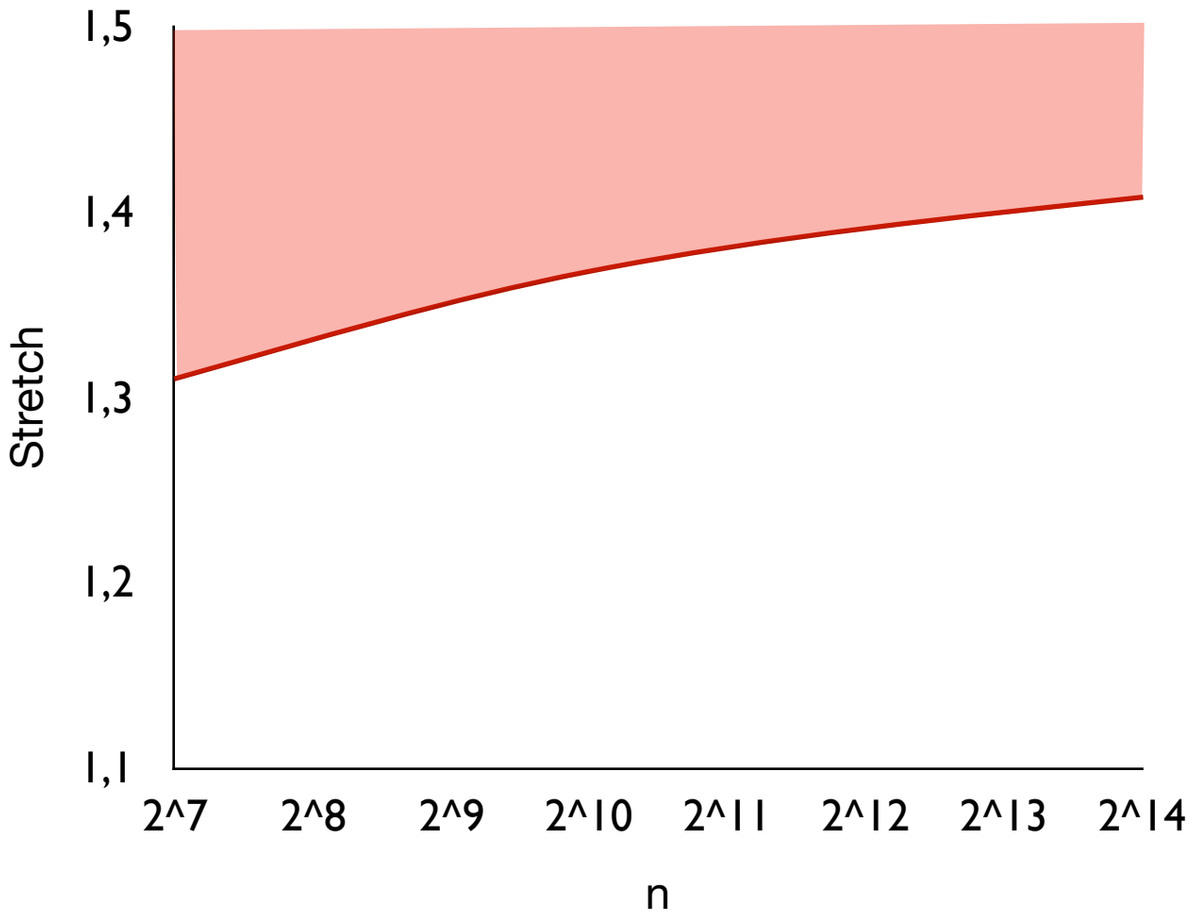

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Experiment

- Verified for small n with Faugère's F4

Gröbner basis approach

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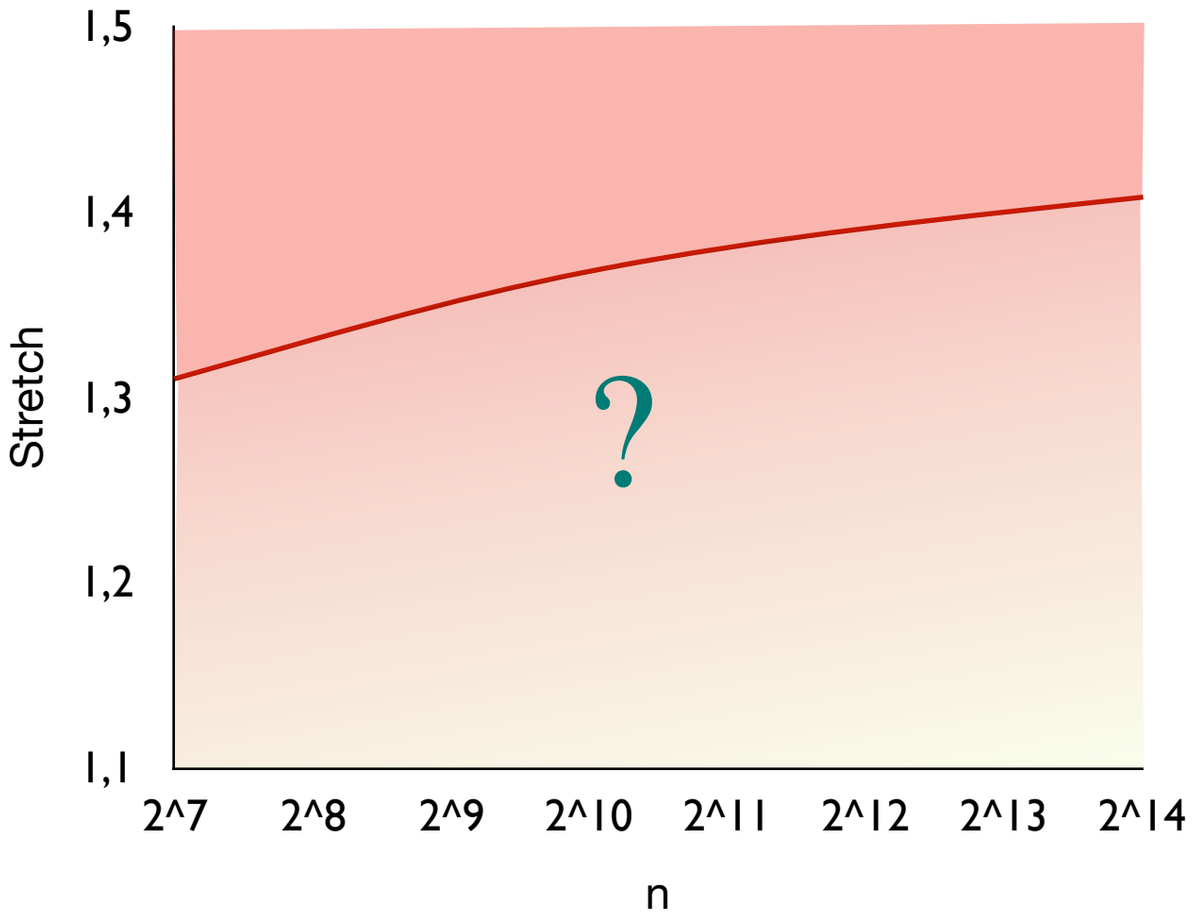
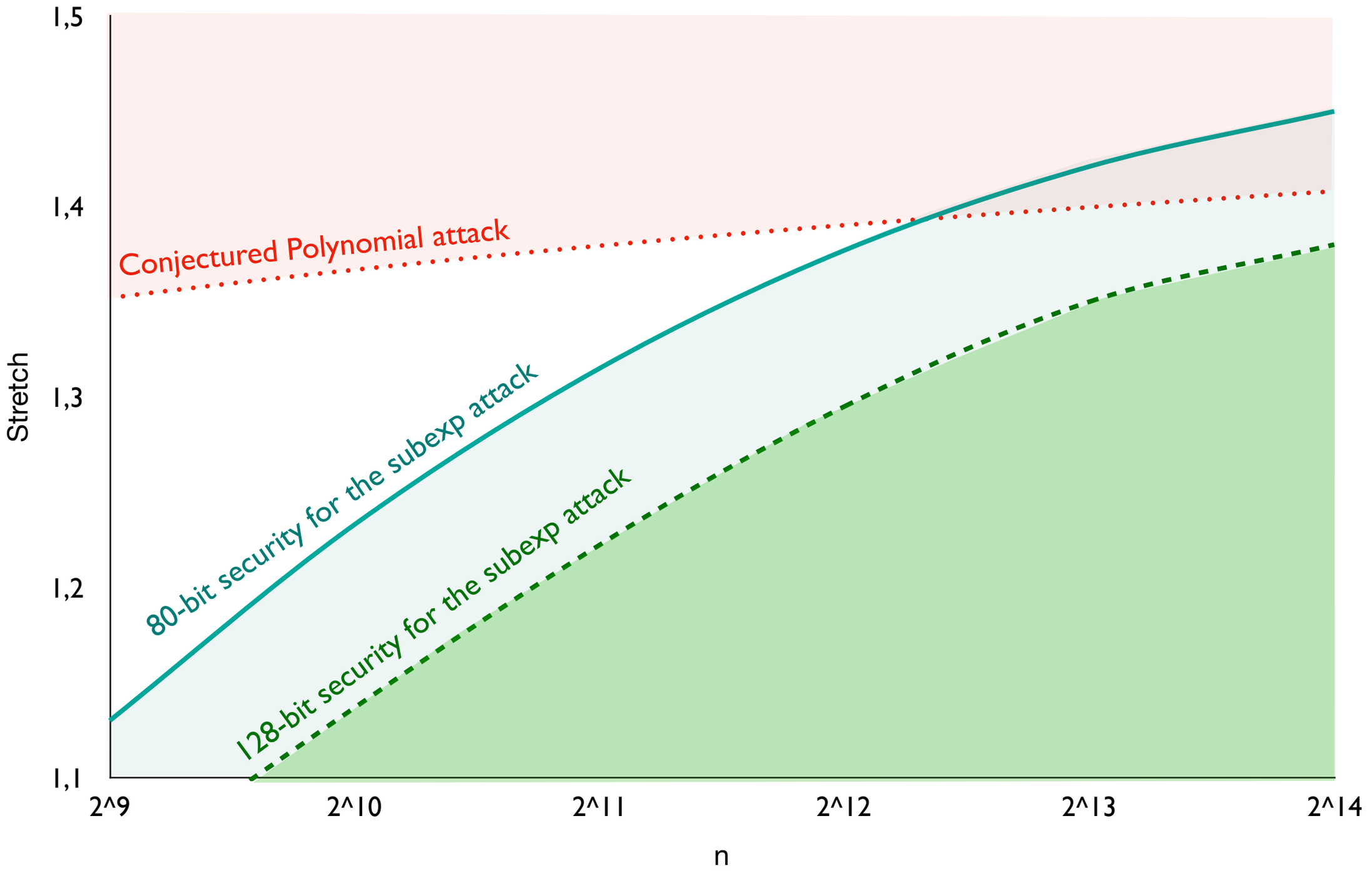


➡ complexity $O(n^{2\omega})$

Experiment

- Verified for small n with Faugère's F4
- Included in the vulnerable parameters

All results


1,5
1,4
1,3
1,2
1,1
Stretch
Conjectured Polynomial attack
80-bit security for the subexp attack
128-bit security for the subexp attack
2^9
2^10
2^11
2^12
2^13
2^14
n

Other contribution

- Our guess and determine attack can be generalized to other predicates:

$$XOR_l MAJ_k(\mathbf{x}) = XOR(x_1, \ldots, x_l) + MAJ(x_{l+1}, \ldots, x_k)$$

XorMaj predicate

Applebaum, Lovett STOC 2016

Other contribution

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$$XOR_l MAJ_k(\mathbf{x}) = XOR(x_1, \ldots, x_l) + MAJ(x_{l+1}, \ldots, x_k)$$

XorMaj predicate

Applebaum, Lovett STOC 2016

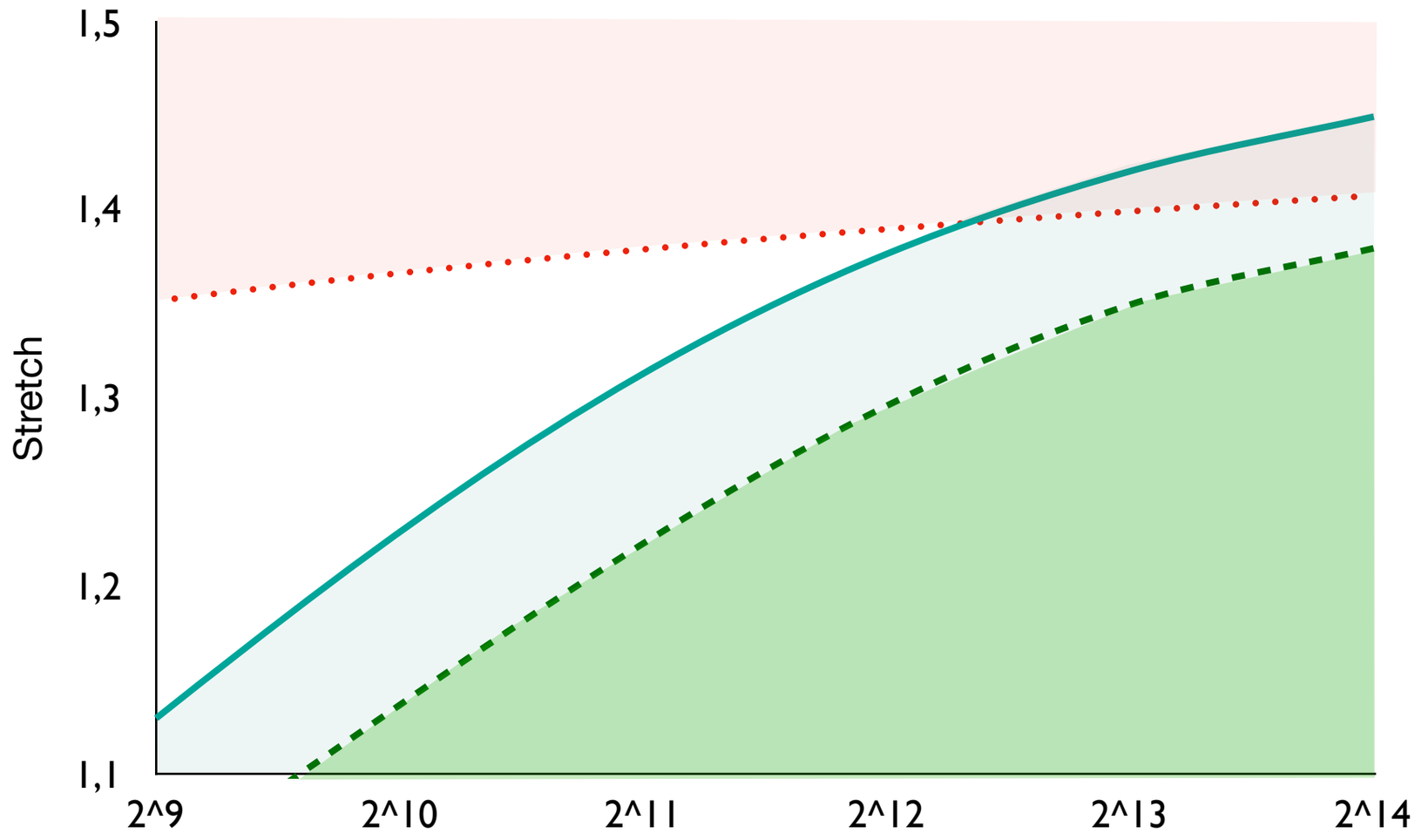
➡ Another approach: The set of guesses is not fixed, and all the guesses are assigned to (0,0,…0) or (1,1,…1)

Total complexity $O\left(n^{\omega} 2^{n\left(1-\frac{s-1}{\left\lceil \frac{k}{2} \right\rceil + 1}\right)}\right)$

Conclusion and open questions

Concrete security of Goldreich PRG with predicate P5 and XorMaj predicates

➡ Can we improve the security bounds for P5?



Conclusion and open questions

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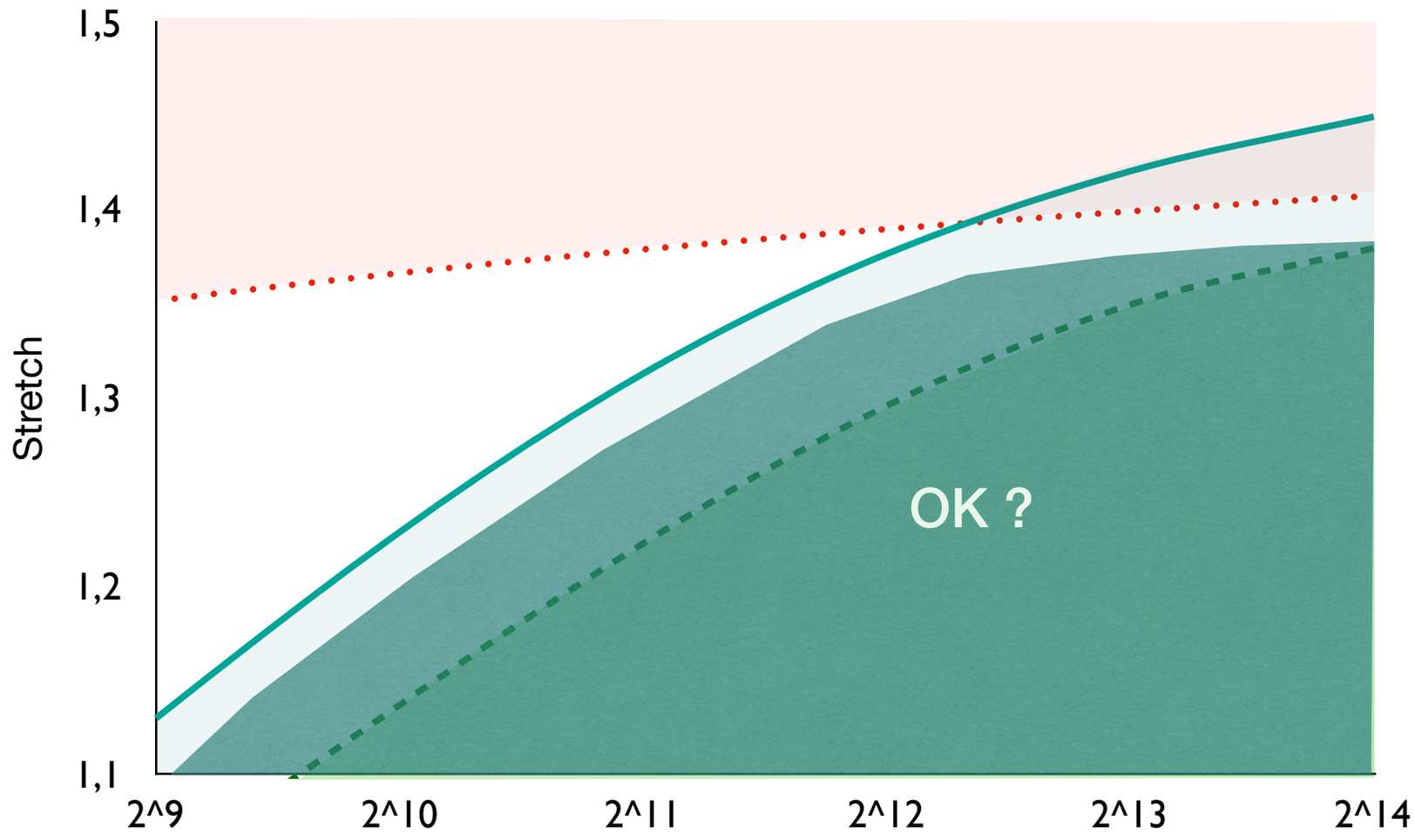

Conclusion and open questions

Concrete security of Goldreich PRG with predicate P5 and XorMaj predicates

➡ Can we improve the security bounds for P5?
for other predicates ?

On other predicates, the inequivalence between the guesses must be taken into account

$x_0 + x_1x_2x_3x_4$ → $x_1 = 0$ Linear equation

$x_0 + x_1x_2x_3x_4$ → $x_1 = 1$ Degree 3 equation

Conclusion and open questions

Eprint: https://eprint.iacr.org/2018/1162

Codes: https://github.com/LuMopY/SecurityGoldreichPRG

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Thank you for your attention