Picnic Post-Quantum Signatures from Zero Knowledge Proofs

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Post-quantum cryptography

A sufficiently powerful quantum computer could factor numbers and compute discrete logarithms

- Breaks essentially all standardized public key crypto
- E.g. RSA, DSA, ECDSA are insecure

Post-quantum cryptography: Design new schemes that

- can be run on classical machines
- Remain secure even if adversary has a quantum computer

Why now? Existing quantum computers only handle a few bits!

- Designing and deploying cryptography is slow!
  - Propose assumptions and schemes
  - Determine candidate parameters
  - Analyze and attack schemes/assumptions
  - Optimize surviving candidates
  - Implement and deploy new schemes
  - Deprecate old algorithms

Post-quantum cryptography

If quantum computers can break factoring and discrete log based crypto, is anything still hard?

Some proposed quantum hard problems:

- Lattice-based problems
- Supersingular isogeny Diffie–Hellman (SIDH)
- Code-based problems
- Multi-variate polynomial problems
- Symmetric key primitives (hash functions, block ciphers)

Post-quantum cryptography

ECDSA gives us small keys, small signatures and fast signing and verification

- But it is insecure against a quantum adversary

Are there any comparable post-quantum proposals?

	Public key size	Signature size	Signing time	Verification time
Lattice (LWE)	Very large	Small	Fast	Fast
Lattice (Ring-LWE)	Large	Small	Fast	Fast
SIDH	Moderate	Large	Very slow	Very slow
Multivariate	Small	Moderate	Moderate	Moderate
Hash (stateful)	Small	Small	Fast	Fast
Hash (stateless)	Small	Moderate	Moderate	Fast

Picnic: Our post-quantum signature scheme

Based on symmetric primitives: a hash function + a block cipher

- Concretely we suggest: SHAKE and LowMC

Efficiency

- Small keys, moderate signature size, moderate signing and verification time

New approach

- Significant opportunity for further optimization
- Diversity of approaches for non-number-theoretic assumptions

Roadmap

Picnic: Basic approach

Picnic: Building blocks

Performance

Picnic 2.0

Conclusion

Picnic: basic approach

Signature from identification scheme (similar to DSA/ECDSA):

Public key = F(sk)

Signature= proof of knowledge of sk (using message as nonce)

- *Proof must not leak sk, so we need a *zero knowledge* proof

For example,
F: hash function

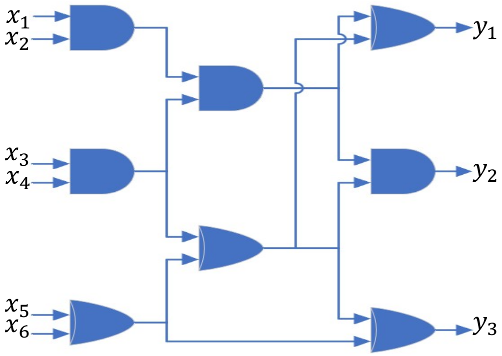
Challenge: we need a **hard to invert function F**, and a **zero knowledge proof system**

- Both need to be secure against quantum adversary

Picnic building blocks: ZKBoo

ZKBoo [GMO16]: zero knowledge proofs for statements about circuits.

Public circuit with AND and XOR gates



Prover wants to prove he knows $x_1 \ldots x_n$ such that the circuit evaluates to $y_1 \ldots y_m$

Signer

sk

Hard to invert F

pk

Built on hash functions and PRNG

Cost depends on the number of AND gates in the circuit and security level

Picnic building blocks: ZKBoo (intuition)

Obviously trivial: just a toy example!

<u>A toy example:</u> Prover wants to prove knowledge of a, b such that $a \oplus b = c$

Prover:

- Step 1: XOR secret share inputs
  - Pick random bits a_1, a_2 that XOR to a and b_1, b_2 for b
  - $\boldsymbol{a_1} \oplus \boldsymbol{a_2} = \boldsymbol{a}$, $\boldsymbol{b_1} \oplus \boldsymbol{b_2} = \boldsymbol{b}$
- Step 2: compute output shares for $\oplus$ gate
  - $\boldsymbol{c_1} = \boldsymbol{a_1} \oplus \boldsymbol{b_1}, \boldsymbol{c_2} = \boldsymbol{a_2} \oplus \boldsymbol{b_2}$
- Step 3: commit to shares
  - Pick random strings r_1, r_2
  - Compute $h_1 = H(\boldsymbol{a_1}, \boldsymbol{b_1}, r_1)$, $h_2 = H(\boldsymbol{a_2}, \boldsymbol{b_2}, r_2)$

Prover → Verifier: c_1, c_2, h_1, h_2

Verifier → Prover: 1

Prover → Verifier: $\boldsymbol{a_1}, \boldsymbol{b_1}, r_1$

Verifier:

- Step 4: Pick 1 or 2 at random
- Step 5:

  Check that $c_1 \oplus c_2 = c$ and $\boldsymbol{a_1} \oplus \boldsymbol{b_1} = c_1$

  Check that $h_1 = H(a_1, b_1, r_1)$

Why is this convincing?

- If Prover computes h_1, h_2 using a_1, a_2, b_1, b_2 such that $a_1 \oplus b_1 = c_1$, $a_2 \oplus b_2 = c_2$, and $c_1 \oplus c_2 = c$ we're done:
  - $(\boldsymbol{a_1} \oplus \boldsymbol{a_2}) \oplus (\boldsymbol{b_1} \oplus \boldsymbol{b_2}) = (\boldsymbol{a_1} \oplus \boldsymbol{b_1}) \oplus (\boldsymbol{a_2} \oplus \boldsymbol{b_2}) = \boldsymbol{c_1} \oplus \boldsymbol{c_2} = \boldsymbol{c}$
- If not, Prover gets caught with probability at least 1/2

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Why does this hide a, b?

- Verifier gets to see:
  - a_1, b_1: reveals no information about a, b
  - $c_1 = a_1 \oplus b_1$, $c_2 = c \oplus c_1$,
  - h_2: hash of randomized inputs

Picnic building blocks: ZKBoo (intuition)

Decrease cheating probability

- Run t copies of proof with fresh randomness, verifier picks a challenge for each
- Probability of cheating decreases exponentially. $(1/3^t)$

Eliminate interaction

- Fiat-Shamir: Choose challenge by hashing (c_1, c_2, h_1, h_2) from all copies.
- If $1/3^t$ is small enough, cheating prover can try hashing many sets of messages, will never find one he can correctly respond to
- Also include signature message in the hash.

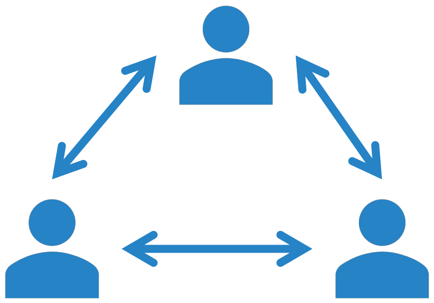
What if we want a circuit with

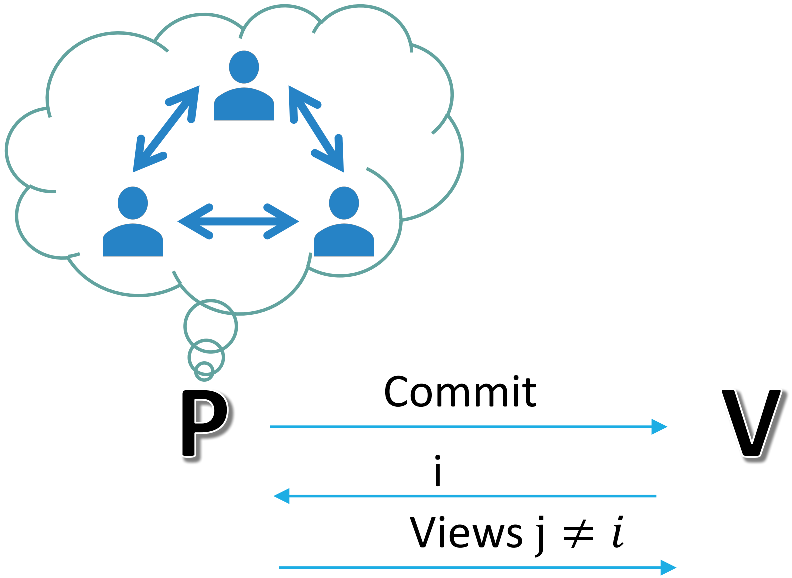
- ANDs
- More gates?

Picnic building blocks: ZKBoo

Foundation for ZKBoo: MPC-in-the-head [IKOS07]

- Approach for constructing ZK proofs from Multi Party Computation
- Multi Party Computation
  - N parties with private input x_i
  - Want to compute f$(x_1, \ldots, x_n)$
  - Even if $n-1$ parties combine their information, they learn nothing else
- To prove "I know x such that F(x)=1"
  - Choose random values such that $x_1 \oplus \cdots \oplus x_n = x$
  - Imagine N parties each with input x_i.
  - Internally run MPC between them to compute $F(x_1 \oplus \cdots \oplus x_n)$.
  - Record all messages sent and received.
  - For each party commit to "view":
    - input x_i, randomness, messages sent, messages received
  - Verifier chooses i
  - Prover reveals views for all parties except i





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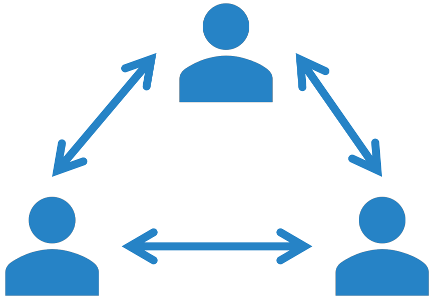

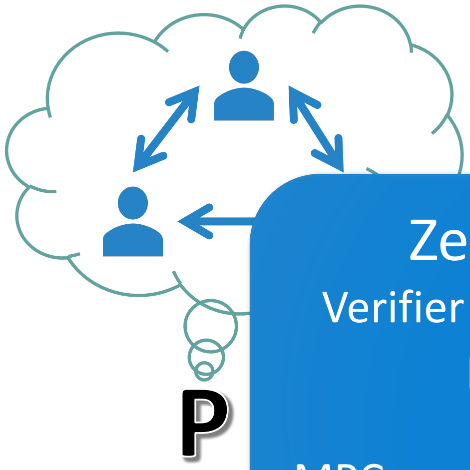

Zero Knowledge

Verifier gets to see views of all parties except i

MPC guarantees it learns nothing besides F(x)

Picnic building blocks: ZKBoo

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  - N parties with private input x_i
  - Want to compute f$(x_1, \ldots, x_n)$
  - Even if $n-1$ parties combine their information, they learn nothing else
- To prove "I know x such that F(x)=y"
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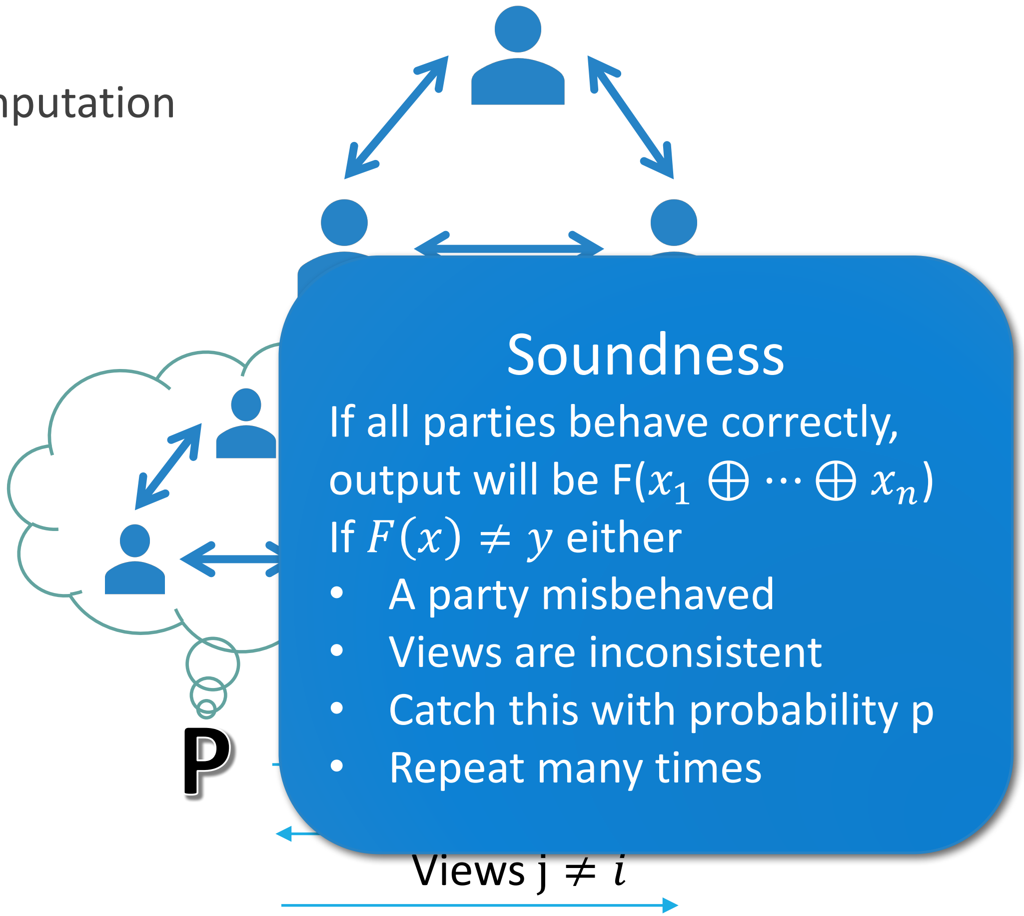

Soundness

If all parties behave correctly, output will be F$(x_1 \oplus \cdots \oplus x_n)$

If $F(x) \neq y$ either

- A party misbehaved
- Views are inconsistent
- Catch this with probability p
- Repeat many times

Picnic building blocks: ZKBoo (intuition)

Obviously trivial: just a toy example!

A toy example: Prover wants to prove knowledge of a, b such that $a \oplus b = c$

Inputs
P_1: $\boldsymbol{a_1}, \boldsymbol{b_1}$
P_2: $\boldsymbol{a_2}, \boldsymbol{b_2}$

MPC

Prover:

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  - Pick random bits a_1, a_2 that XOR to a and b_1, b_2 for b
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P_1 P_2

Prover → Verifier: c_1, c_2, h_1, h_2

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Inputs
P_1: $\boldsymbol{a_1}, \boldsymbol{b_1}$
P_2: $\boldsymbol{a_2}, \boldsymbol{b_2}$

MPC

Commit to views

P_1

P_2

Prover → Verifier: c_1, c_2, h_1, h_2

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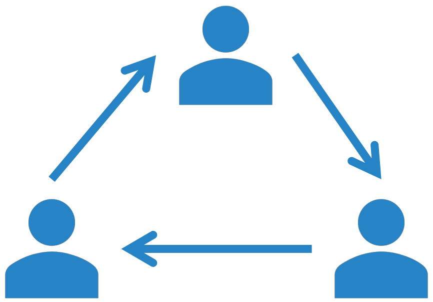
  Check that $h_1 = H(a_1, b_1, r_1)$

Check P_1's work

Picnic building blocks: ZKBoo

ZKBoo makes MPC-in-the-head practical

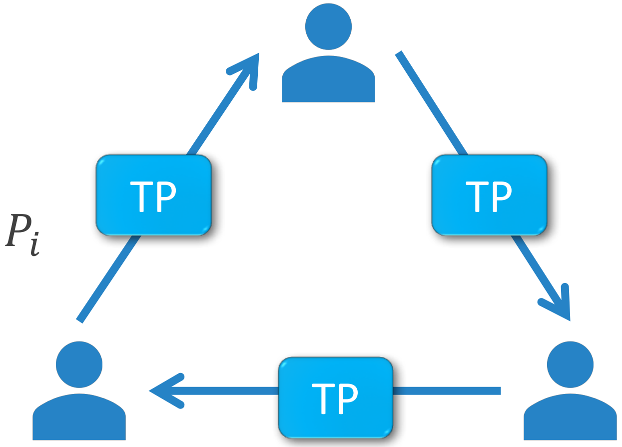


Minimize communication

- Fix 3 parties (in general commication is n^2)
- P_i only receives messages from P_{i+1}

Observation :

- we said V checks that messages sent = messages received
- Instead, could check any function on views of P_i and P_{i+1} up to that point
- Message received can be function of current state of P_{i+1} and previous state of P_i
- Optimize MPC in this model



Picnic building blocks: ZKB++

ZKB++: Optimized ZKBoo [CDGORRSZ17]

- Identify places where e.g. values can safely be recomputed by the verifier, or represented by a short seed
- Reduces signature size by more than factor of 2
- Security analysis in random oracle model

Variant based on Unruh's transform [Unruh 15]

- Security analysis in quantum random oracle model
- Our optimized implementation increases signature size by 1.6x over basic ZKBoo++
  - Still shorter than original ZKBoo

Picnic: basic approach

Signature from identification scheme (similar to DSA/ECDSA):

Public key = F(sk)

Signature= proof of knowledge of sk (using message as nonce)

- *Proof must not leak sk, so we need a *zero knowledge* proof

For example,
F: hash function

Challenge: we need a **hard to invert function F**, and a **zero knowledge proof system**

- Both need to be secure against quantum adversary

Picnic building blocks: choosing F

ZKBoo++: Prover/signer can prove he knows sk such that the circuit F evaluates to pk

What F should we choose?

- F must be hard to invert
- Proof/signature size depends on number of AND gates in circuit for F

We can use a block cipher as well:

- PK: $R, Enc_{sk}(R)$

	Sec level	AND gates
AES	128	5440
SHA-2	256	> 25000
SHA-3	256	38400
Noekeon	128	2048
Trivium	80	1536
PRINCE		1920
Fantomas	128	2112
Kreyvium	128	1536
FLIP	128	> 100000
MIMC	128	10337
MIMC	256	41349
LowMC	128	< 800
LowMC	256	< 1400

Picnic building blocks: LowMC

New block cipher introduced by [*ARSTZ15*]

LowMCv2: updated version (eprint16)

Substitution-permutation-network design

Parameterizable:

- allows for minimizing AND gates or AND depth
- Tradeoffs between #s of AND gates and XOR gates
- Variable key and block sizes
- Allows for different security levels and #of plaintext ciphertext pairs the attacker will be given

For our application

- Few (but not minimal) AND gates: balance signature size and signing time

Picnic building blocks: LowMC

New block cipher introduced by [*ARSTZ15*]

LowMCv2: updated version (eprint16)

Substitution-permutation-network design

Security for our application

- Several different security levels based on desired security for signature
- Only 1 plaintext-ciphertext pair is revealed
- Keysize = blocksize
- Attackers goal is key recovery*
- Weaker than traditional indistinguishable security with many plaintext-ciphertext pairs
- Our parameters may be conservative

Roadmap

Picnic: Basic approach

Picnic: Building blocks

Performance

Picnic 2.0

Conclusion

Picnic 1.0 Performance

3 parameter levels

- L1: 128 bits classical, 64 bits quantum
- L3: 192 bits classical, 96 bits quantum
- L5: 256 bits classical, 128 bits quantum

LowMC parameters
# of repetitions

Signature and key sizes (bytes)

Parameter Set	Public Key	Private Key	Signature
Picnic-L1-FS	32	16	34000
Picnic-L1-UR	32	16	53929
Picnic-L3-FS	48	24	76740
Picnic-L3-UR	48	24	121813
Picnic-L5-FS	64	32	132824
Picnic-L5-UR	64	32	209474

Picnic 2.0 has significant improvements

Picnic 1.0 Performance

3 parameter levels

- L1: 128 bits classical, 64 bits quantum
- L3: 192 bits classical, 96 bits quantum
- L5: 256 bits classical, 128 bits quantum

→ LowMC parameters
# of repetitions

Optimized constant- time implementation (ms), Intel(R) Core(TM) i7-4790 CPU @ 3.60GHz

Parameter Set	Keygen	Sign	Verify
Picnic-L1-FS	0.00	5.41	3.70
Picnic-L1-UR	0.00	6.12	4.24
Picnic-L3-FS	0.01	17.07	11.61
Picnic-L3-UR	0.01	19.01	13.08
Picnic-L5-FS	0.02	36.47	24.70
Picnic-L5-UR	0.02	39.21	26.90

Experiments

TLS integration:

- What if we want to use Picnic for TLS authentication?
- Added Picnic to the *Open Quantum Safe library* (OQS), the OQS fork of OpenSSL and Apache web server
- Use Picnic to create X509 certificates certifying Picnic public keys
- Use resulting certificates to establish TLS 1.2 connections

HSM implementation:

- What if a CA wants to store Picnic signing keys in an HSM?
- Experimented with the Utimaco SecurityServer Se50 LAN V4
- Implemented Picnic key generation and signing in an HSM.

See Picnic design document For details

Roadmap

Picnic: Basic approach

Picnic: Building blocks

Performance

Picnic 2.0

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Picnic 2.0 building blocks: [KKW18 proofs]

[KKW18] introduced an improved proof system

- ZKBoo soundness for 1-round: 1/3 because we fully check 1 party of 3.
- What if we could fully check n-1 out of n?
- We could run fewer parallel repetitions!
- Need to guarantee:
  - We can check each opened parties
  - We can increase the number of parties without increasing communication
  - We can regenerate n-1 views from little information
- Use MPC in the preprocessing model
  - Commit to preprocessing, and use cut-and-choose to check
  - Protocol just has 1 broadcast bit/AND gate from each party
  - Just need to send broadcast bits from unopened party
- Picnic 2.0 uses 64 parties, checks 63.
- Improves signature size by almost a factor of 3

Need to make sure this communication is small: clever tree data structure

Picnic 2.0 building blocks: [KKW18] proofs

Signatures sizes for Picnic with [KKW18] proofs

Security Level	Previous Size (bytes)	New Size (bytes)	
L1-FS	32,838	12,359	2.7x
L3-FS	74,134	27,172	2.7x
L5-FS	128,176	46,282	2.8x

- Sizes given are the average case sizes
- The implementation from ePrint 2018/475 is suggests it's possible to have the same performance
- The parameters using the Unruh transform are unchanged

Picnic 2.0 building blocks: Optimized LowMC

[KPPRR17, D18]

LowMC was designed to support arbitrary parameter sets (key size, block size, # rounds, # s-boxes)

This work optimizes for the Picnic parameters:

- LowMC is an SPN cipher
- rounds have a s-box (nonlinear) part and a linear part
- Picnic: small nonlinear part and a large linear part
- Reorder operations to combine some linear steps

Gives faster signing/verification by factor of ~2-3.

Picnic 2.0 building blocks: Optimized LowMC

Running times with optimized LowMC circuit

Parameters	Sign (ms, old)	Sign (ms, new)		Verify (ms, old)	Verify (new)	
L1-FS	5.41	2.37	2.28x	3.70	1.89	1.96x
L1-UR	6.12	3.08	1.99x	4.24	2.47	1.72x
L3-FS	17.07	5.50	3.10x	11.61	4.49	2.59x
L3-UR	19.01	7.43	2.56x	13.08	5.98	2.19x
L5-FS	36.47	9.74	3.74x	24.70	8.05	3.07x
L5-UR	39.21	12.58	3.12x	26.90	10.25	2.62x

- This compares versions of the constant time implementations
- Times are milliseconds on an Intel Core i7-4790 CPU @ 3.60GHz
- Does not include [KKW18] proofs

Picnic 2.0 building blocks: Optimized LowMC

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Conclusions

New postquantum signature proposal

- Based on symmetric primitives: a hash function + hard-to-invert function (concretely SHAKE and LowMC)
- Small keys, moderate signature size, moderate signing and verification time
- Modular construction from ZK proofs

Lots of opportunity for further optimization

- Further optimize current proof system?
- Further design of MPC protocols for this setting?
- Propose new proof system (sublinear proofs?)
  - Ligero [AHIV17] is work in this direction
- Further optimizations for LowMC?
- Security analysis of LowMC for our parameters
- Or alternative functions F?

More info, see https://microsoft.github.io/Picnic/ . Picnic 2.0 parameters and code available later this week.