

Signatures with Flexible Public Key: Introducing Equivalence Classes for Public Keys

Michael Backes, Lucjan Hanzlik, Kamil Kluzcniak, **Jonas Schneider**



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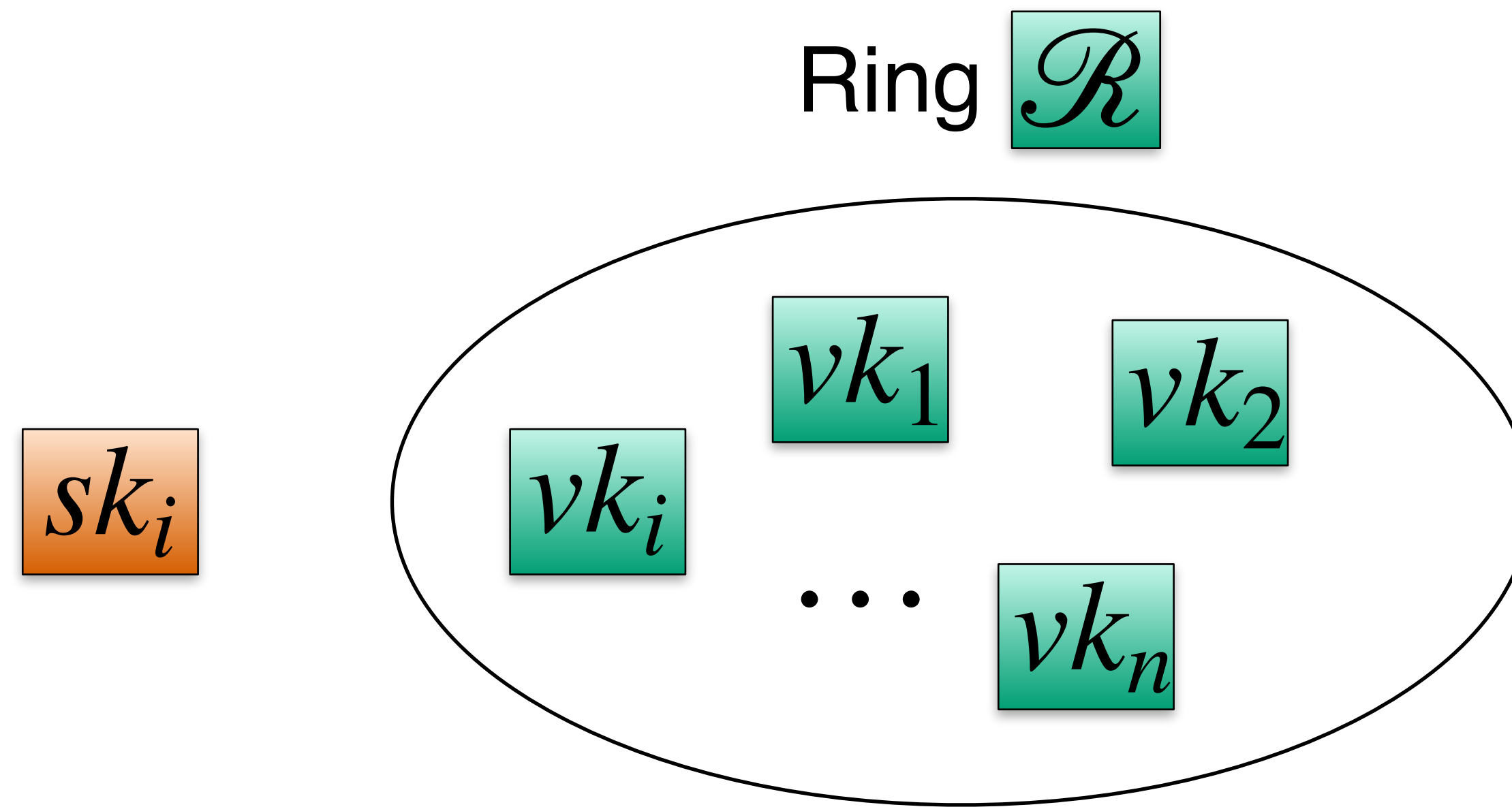


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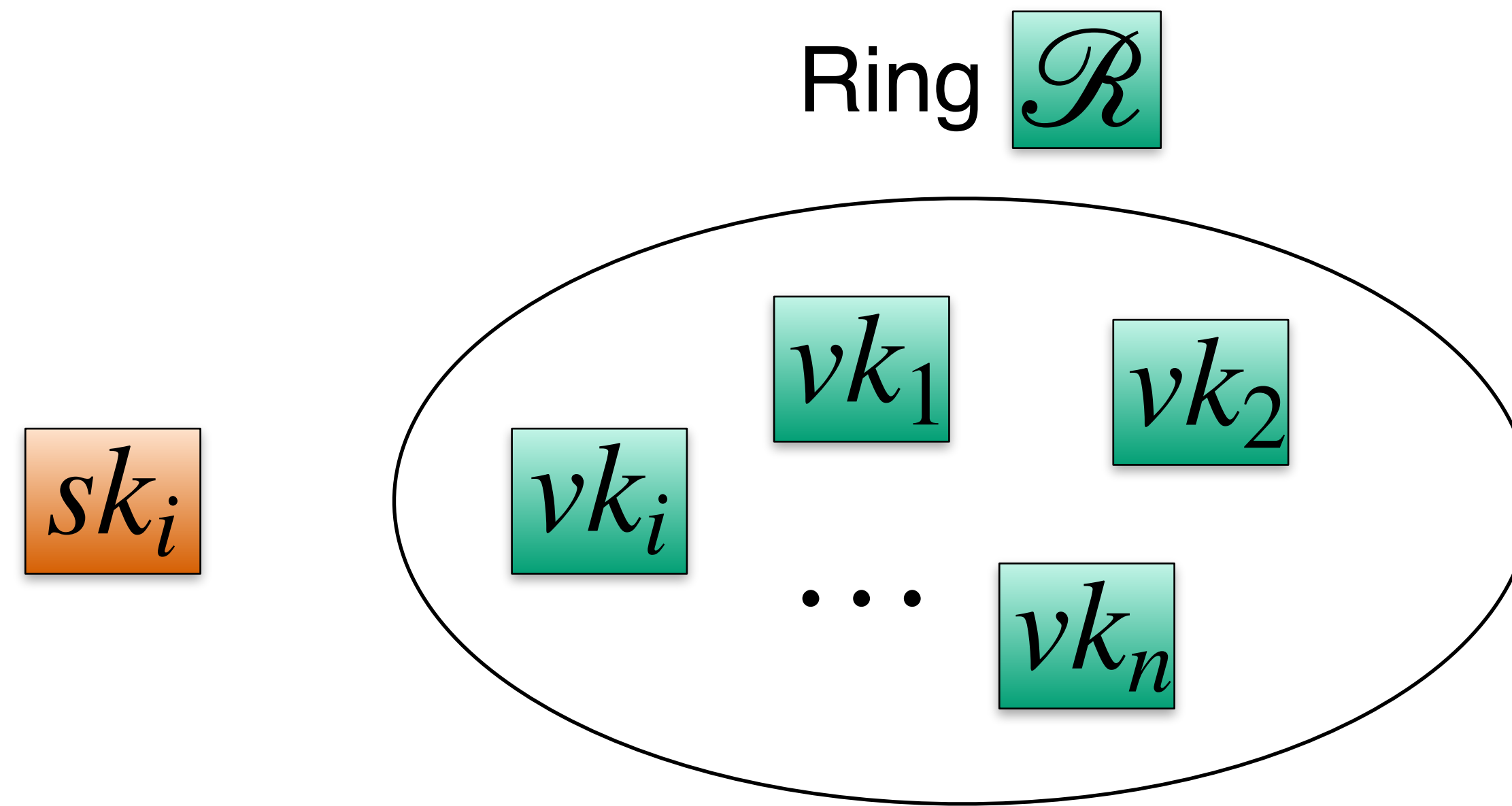
- New primitive! Signatures with Flexible Public Key
 - Applications to
 - sub-linear size Ring Signatures from *falsifiable assumptions without trusted setup*
 - efficient standard model Group Signatures in combination with SPS-EQ

Ring Signatures [Rivest-Shamir-Tauman, 2001]

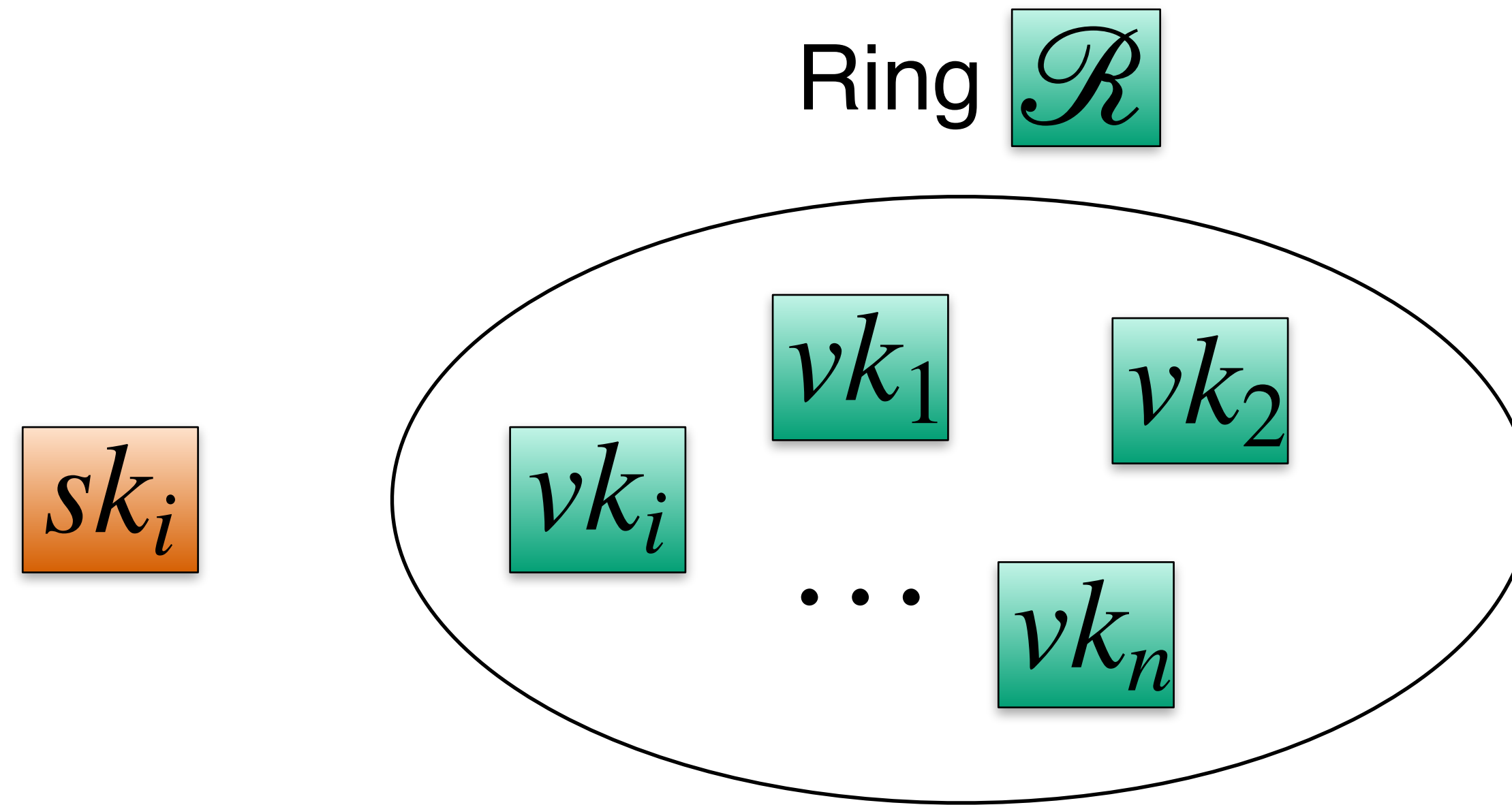
Ring Signatures [Rivest-Shamir-Tauman, 2001]



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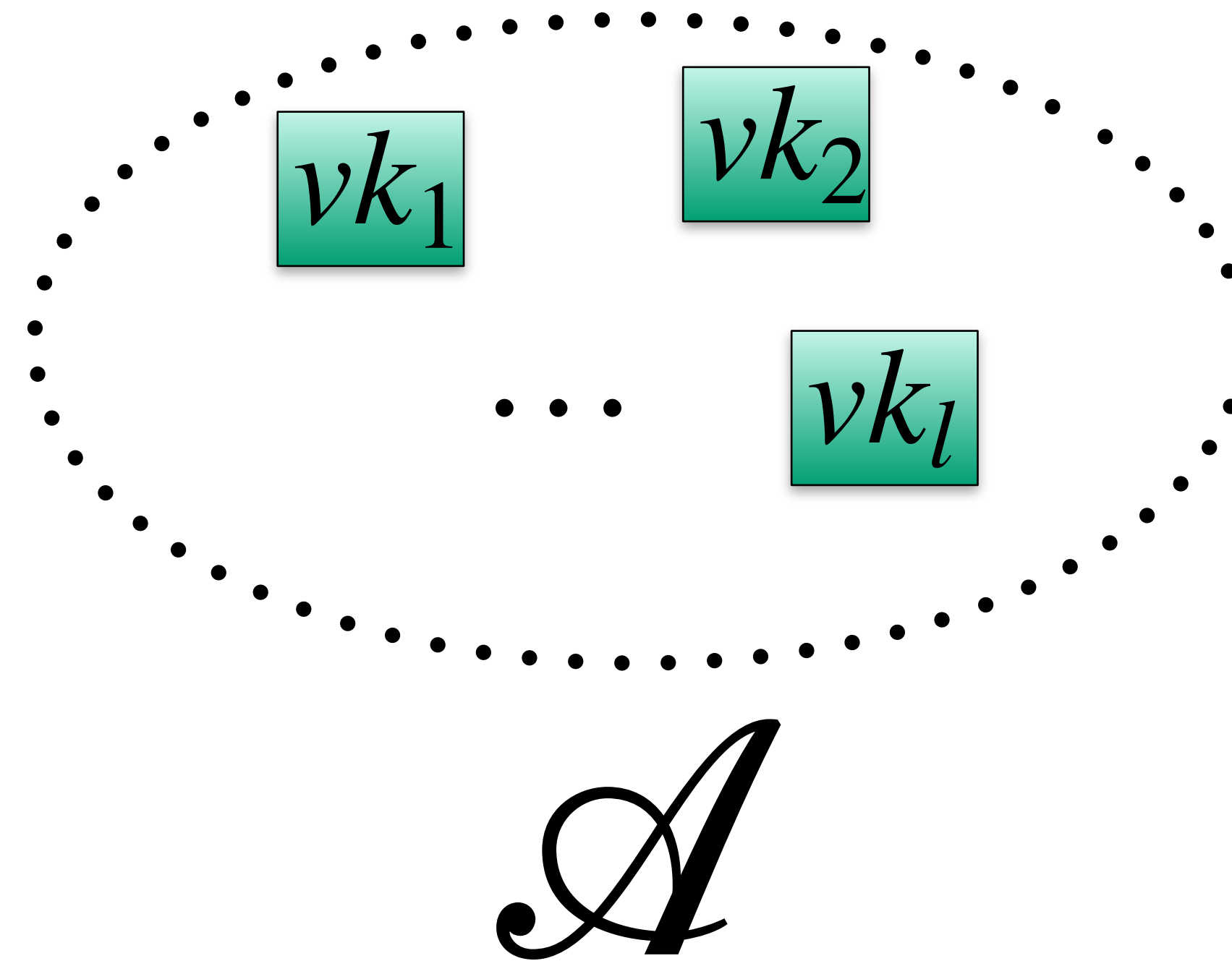
$$\sigma \leftarrow \text{Sign}(sk_i, m, \mathcal{R})$$

$$\text{accept/reject} \leftarrow \text{Verify}(\sigma, m, \mathcal{R})$$

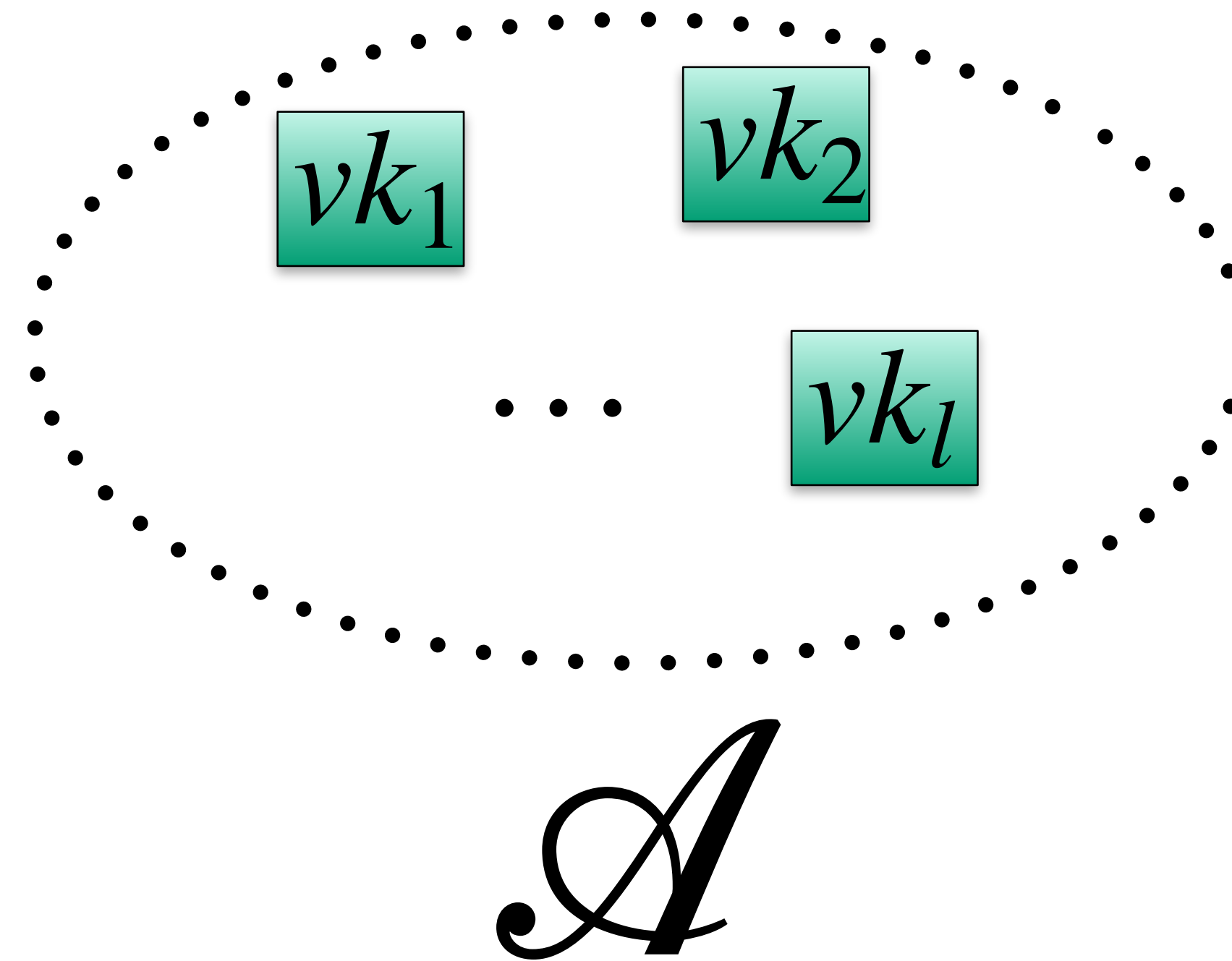
Ring Signatures: Unforgeability [Bender-Katz-Morselli, 2006]

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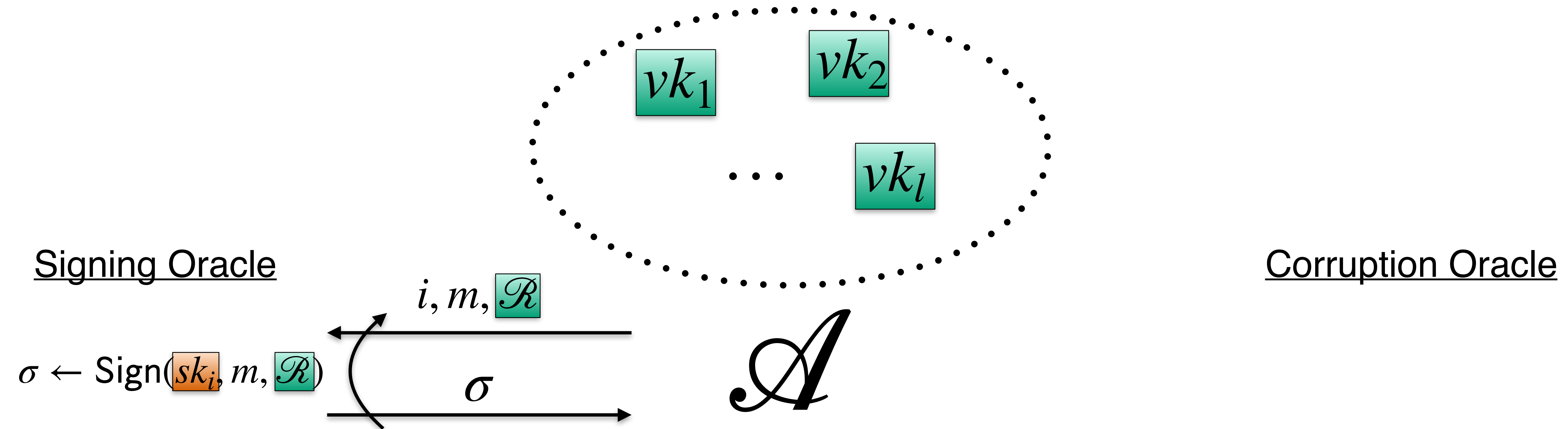
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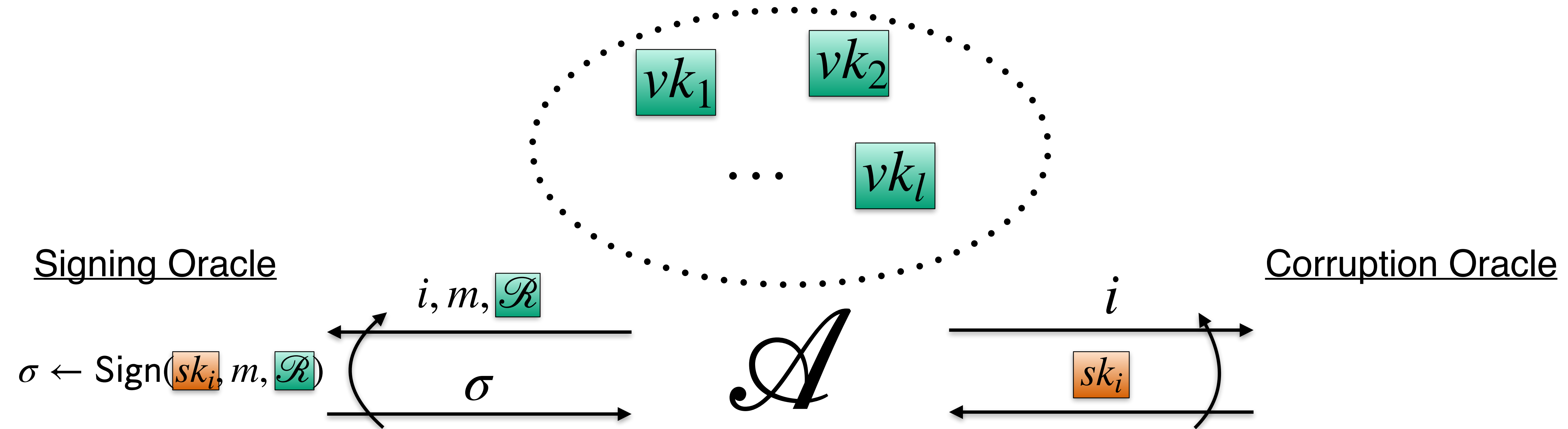
Signing Oracle

Corruption Oracle

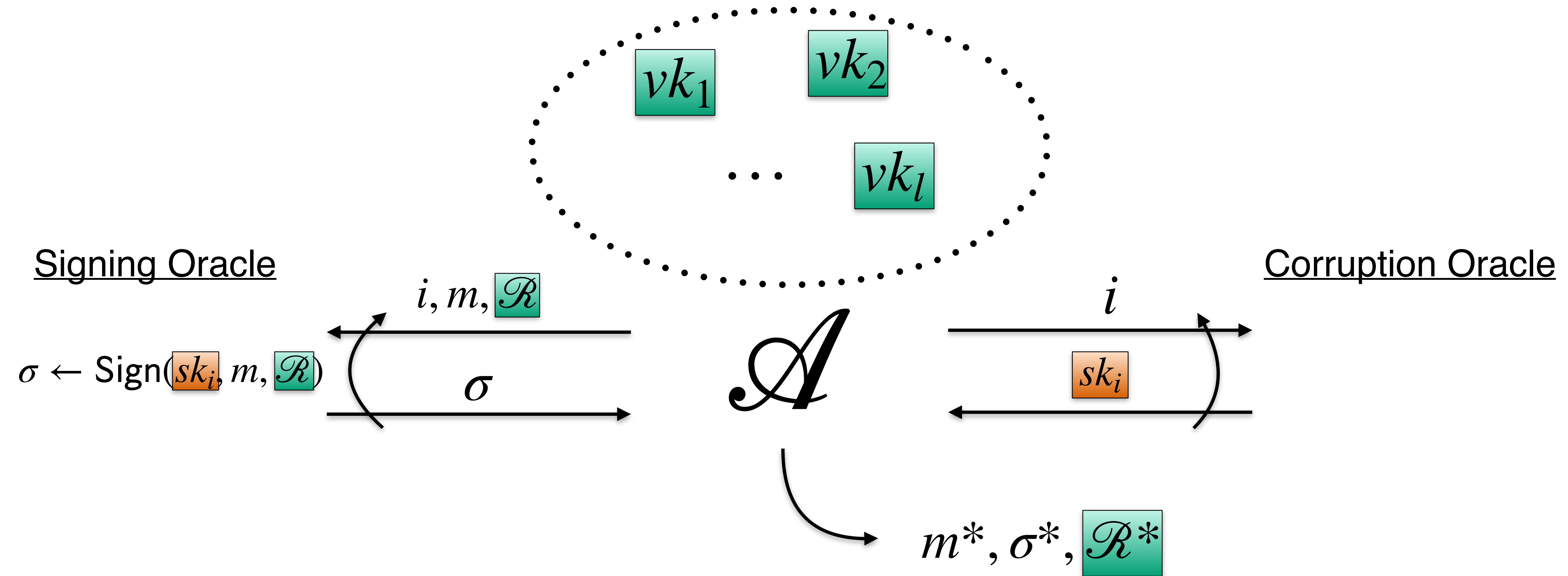
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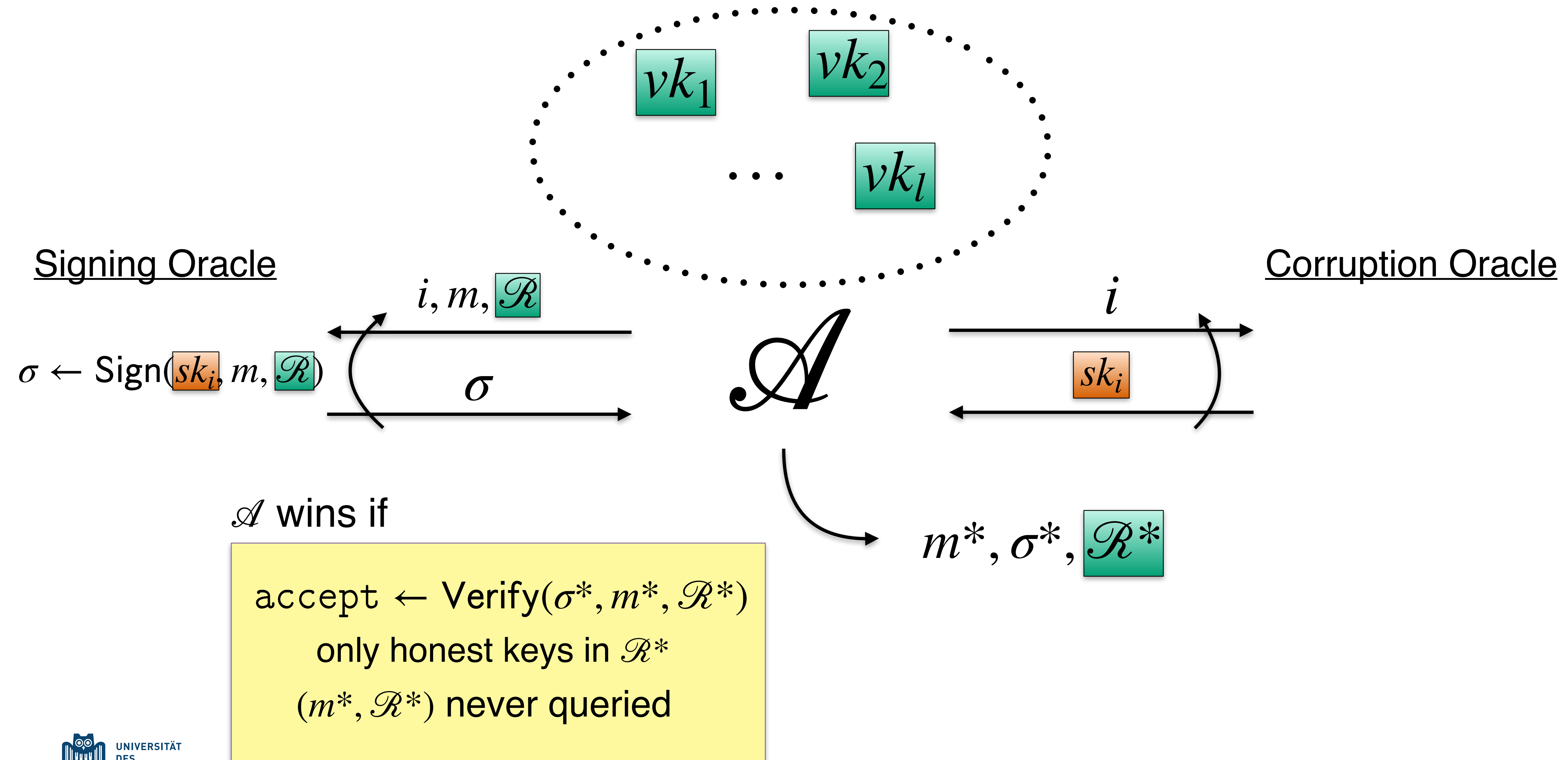
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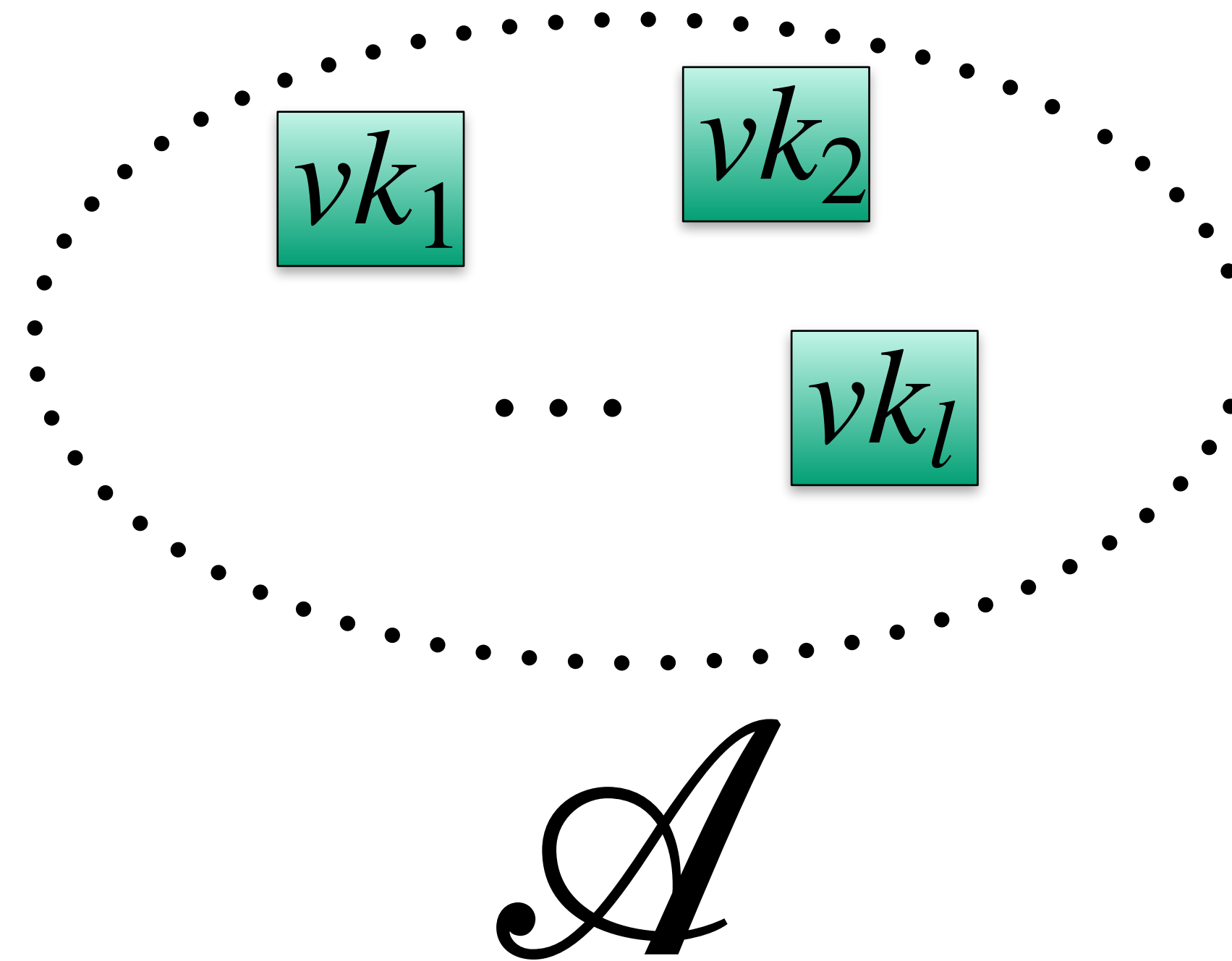
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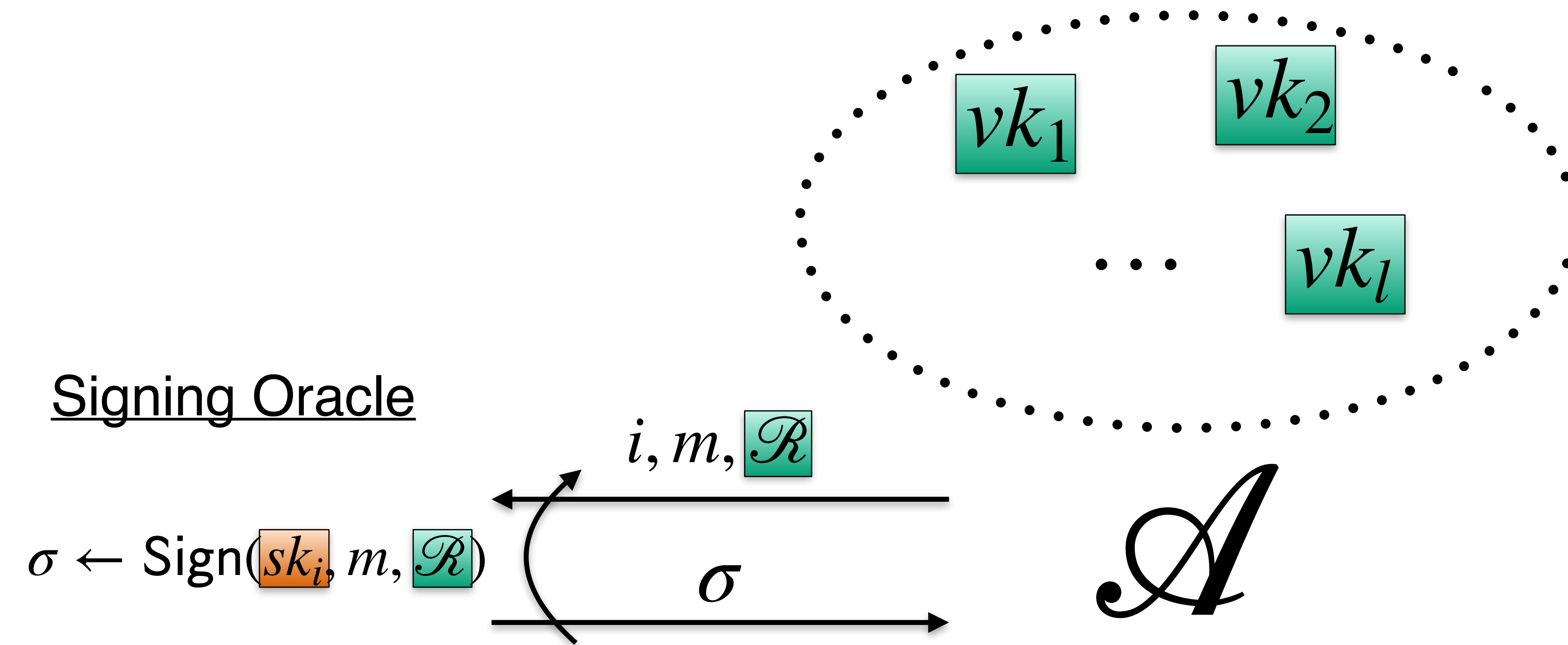
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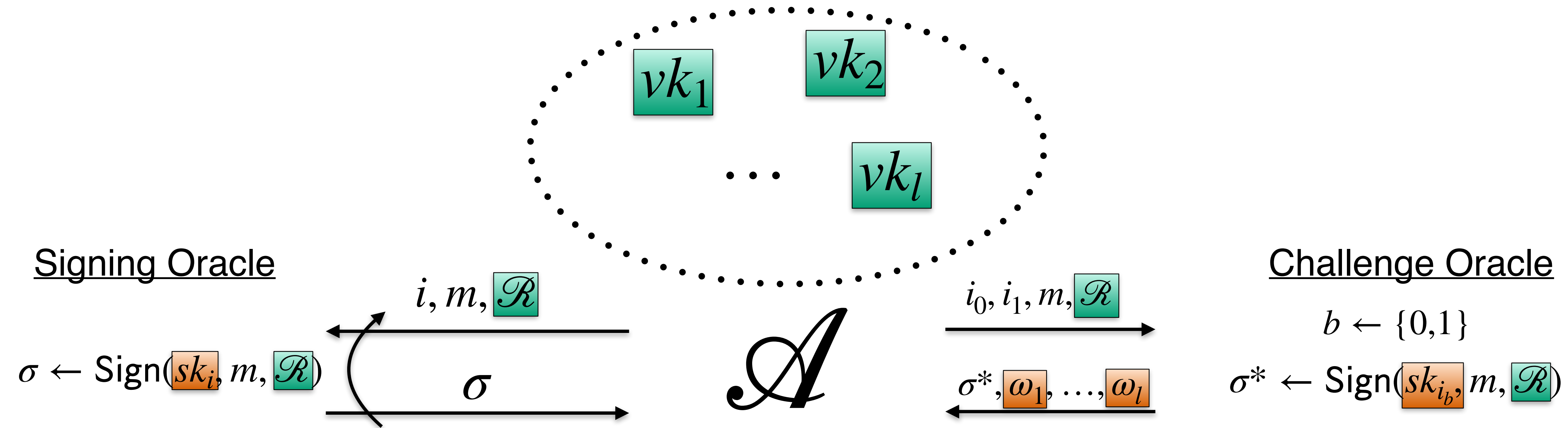
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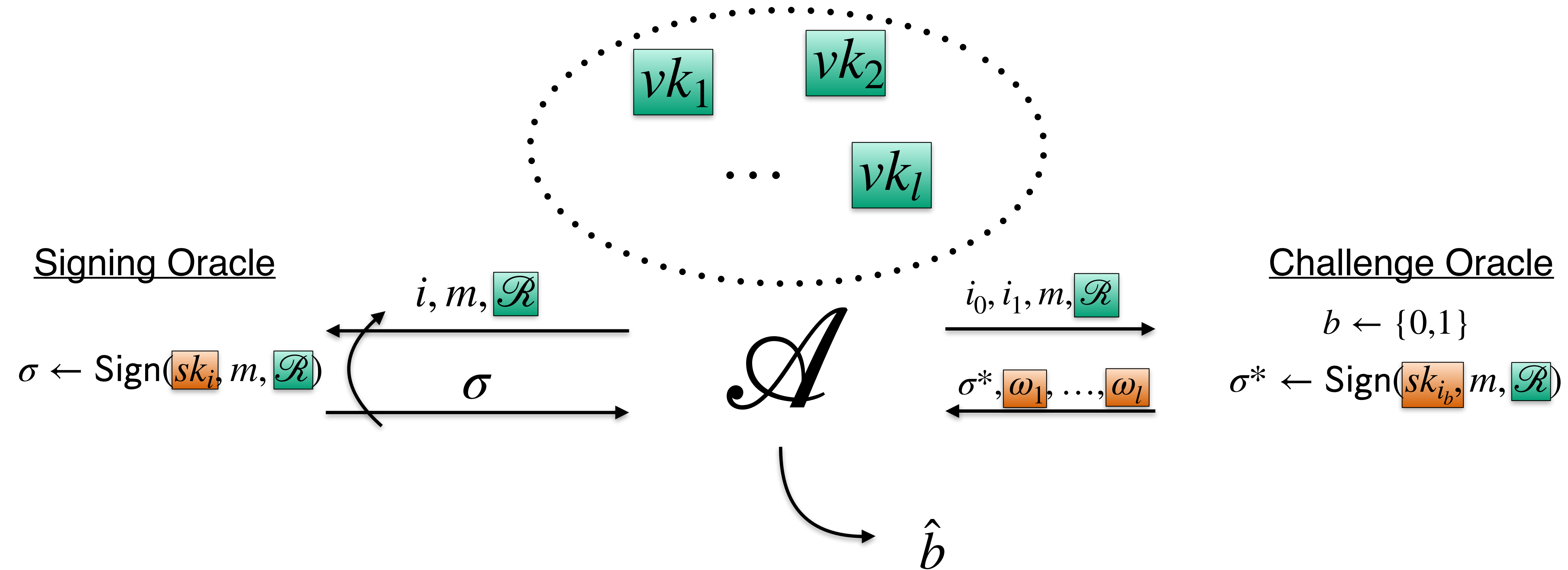
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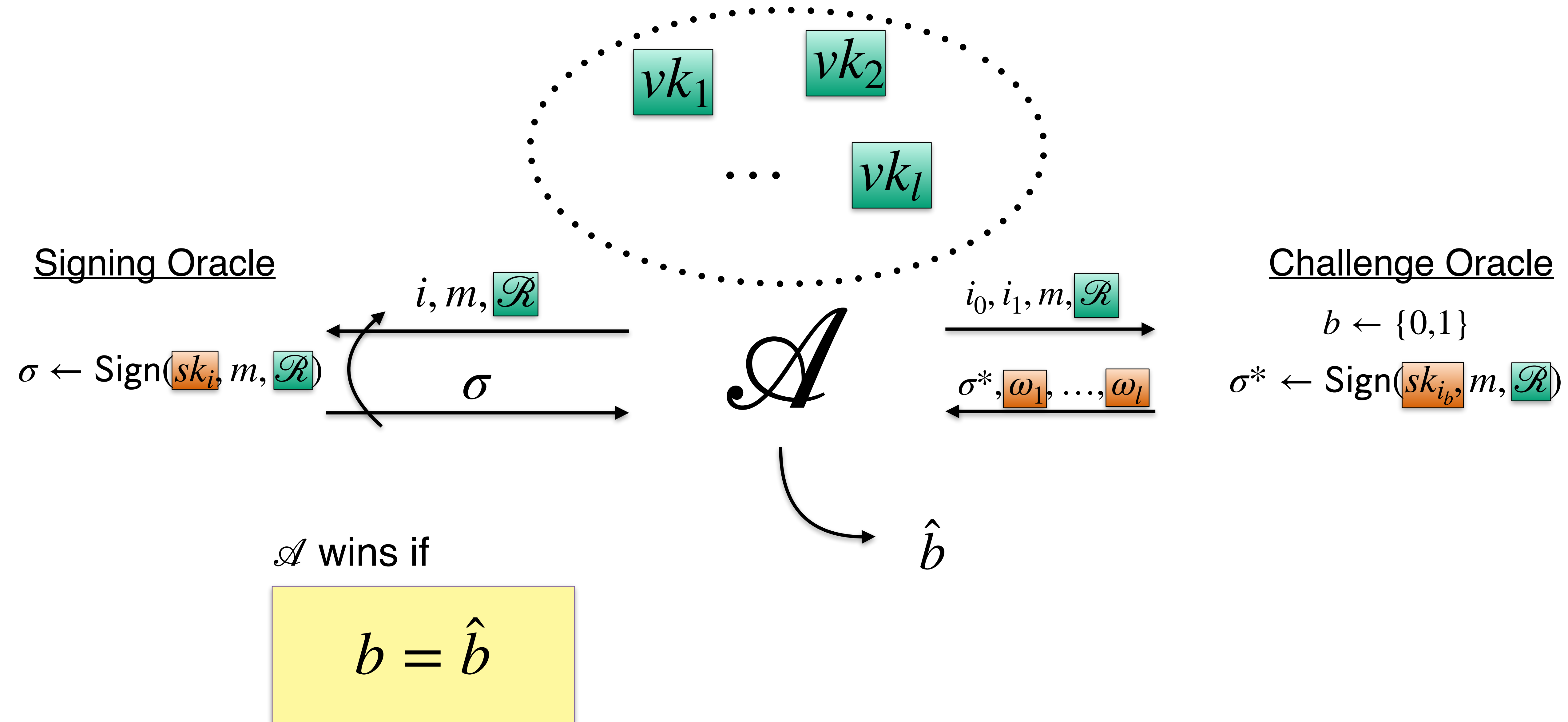
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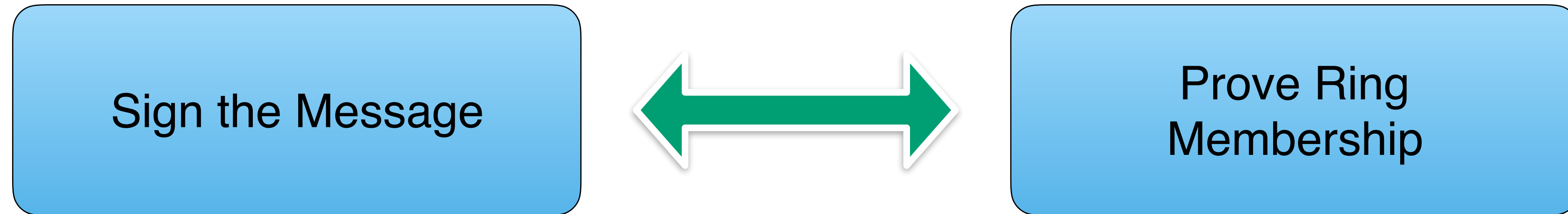
- Standard Model security, falsifiable assumptions, no ROM
- Small signatures for efficiency
- No form of trusted setup!

Ring Signatures: Generic Approach

Sign the Message

Prove Ring
Membership

Ring Signatures: Generic Approach



Ring Signatures: [Malavolta-Schröder, 2017]

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$$\boxed{sk'} \leftarrow \Sigma . \text{RerandSK}(\boxed{sk}, r)$$
$$\sigma_m \leftarrow \Sigma . \text{Sign}(\boxed{sk'}, m \parallel \mathcal{R})$$

Σ : Signature Scheme with Rerandomizable Keys
[Fleischhacker-Krupp-Malavolta-Simkin-S-Schröder, 2016]

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$$\sigma = (\boxed{vk'}, \sigma_m, \pi)$$

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Π : tailored NIZK-PoK with shared setup

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- No Setup
- Security under q-Strong DH assumption + Linear-KEA (GGM)
- $O(n)$ signature size

Signatures with Flexible Public Key (SFPK)

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$$(\boxed{sk}, \boxed{vk}) \leftarrow \text{KeyGen}(1^\lambda)$$

$$\sigma \leftarrow \text{Sign}(\boxed{sk}, m)$$

$$\begin{array}{l} \text{accept} \\ \text{reject} \end{array} \leftarrow \text{Verify}(\boxed{vk}, \sigma, m)$$

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$$\boxed{vk'} \leftarrow \text{ChangeRepVK}(\boxed{vk}, r)$$

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$$(\boxed{\tau}, \boxed{sk}, \boxed{vk}) \leftarrow \text{TKeyGen}(1^\lambda)$$

$$\begin{array}{l} \text{yes} \\ \text{no} \end{array} \leftarrow \text{CheckRep}(\boxed{\tau}, \boxed{vk}, \boxed{vk'})$$

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$$\boxed{vk'} \sim \boxed{vk}$$

Example

$$vk, vk' \in \mathbb{G}^\ell$$

$$vk \sim vk' \text{ if there is } r \in \mathbb{Z}_p^*$$

$$(vk_1^r, \dots, vk_\ell^r) = (vk'_1, \dots, vk'_\ell)$$

SFPK: Unforgeability

A

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$$(\tau, sk, vk) \leftarrow \text{TKeyGen}(1^\lambda)$$



$$(\tau, vk)$$

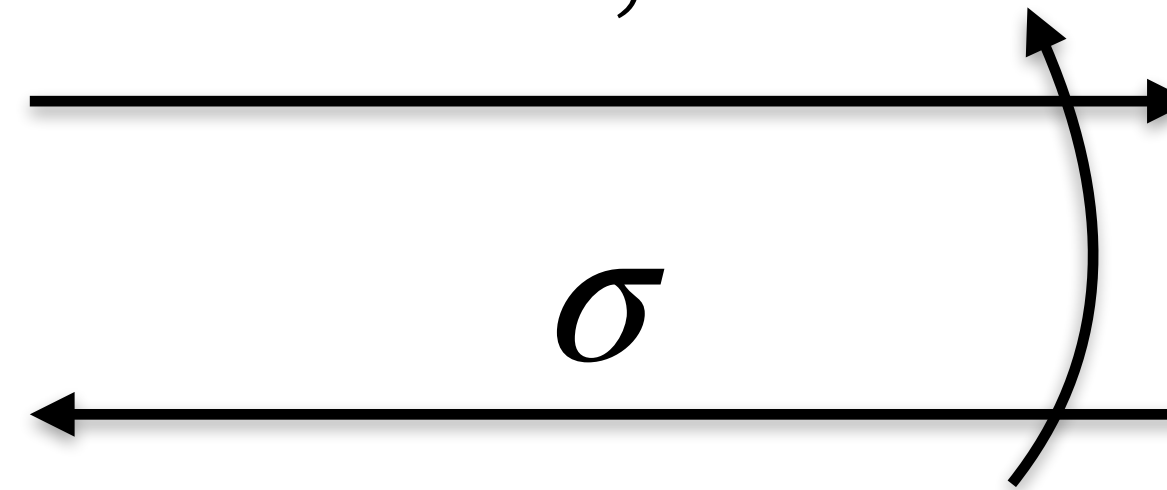
$\mathcal{A}$

$(\tau, sk, vk) \leftarrow \text{TKeyGen}(1^\lambda)$

(τ, vk)

$\mathcal{A}$

m, r



$sk' \leftarrow \text{ChangeRepSK}(sk, r)$

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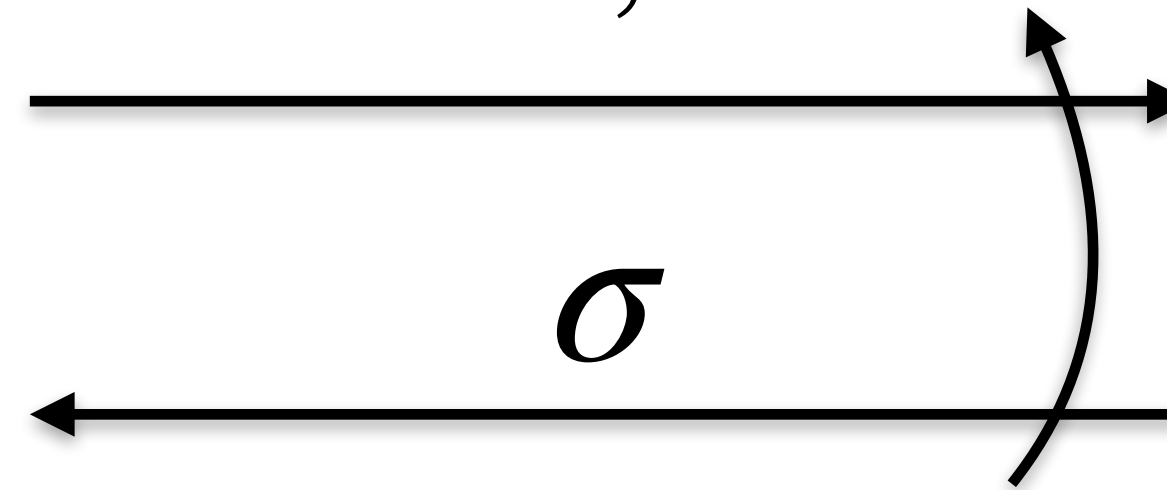
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(vk^*, m^*, σ^*)

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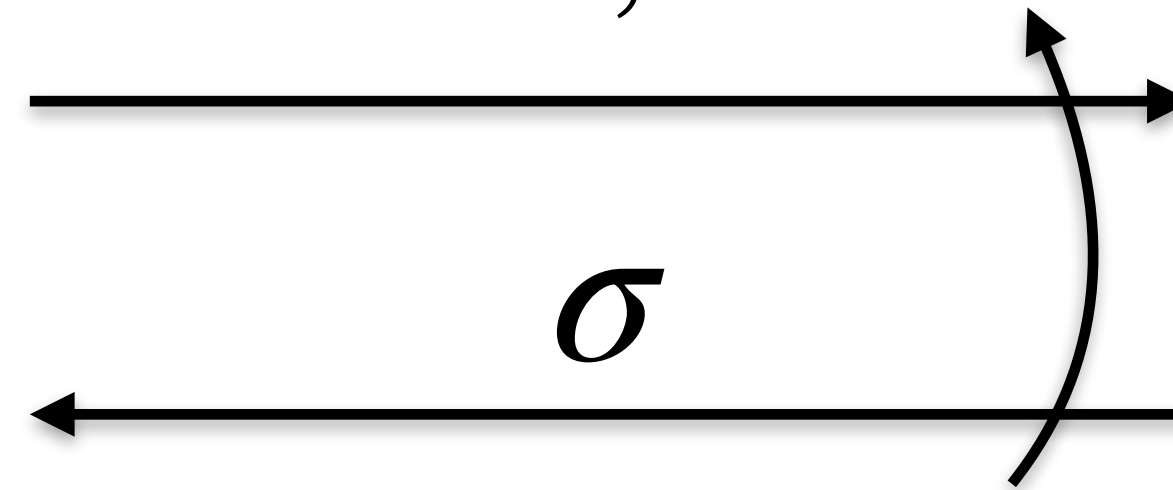
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$\mathcal{A}$

(vk^*, m^*, σ^*)

m, r



$sk' \leftarrow \text{ChangeRepSK}(sk, r)$

$\sigma \leftarrow \text{Sign}(sk', m)$

$\mathcal{A}$ wins if

$\text{accept} \leftarrow \text{Verify}(vk^*, \sigma^*, m^*)$

m^* never queried

$\text{yes} \leftarrow \text{CheckRep}(\tau, vk, vk^*)$

SFPK: Class-Hiding

A

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$$\begin{aligned} (\boxed{sk_0}, \boxed{vk_0}) &\leftarrow \text{KeyGen}(1^\lambda; \boxed{\omega_0}) \\ (\boxed{sk_1}, \boxed{vk_1}) &\leftarrow \text{KeyGen}(1^\lambda; \boxed{\omega_1}) \end{aligned}$$

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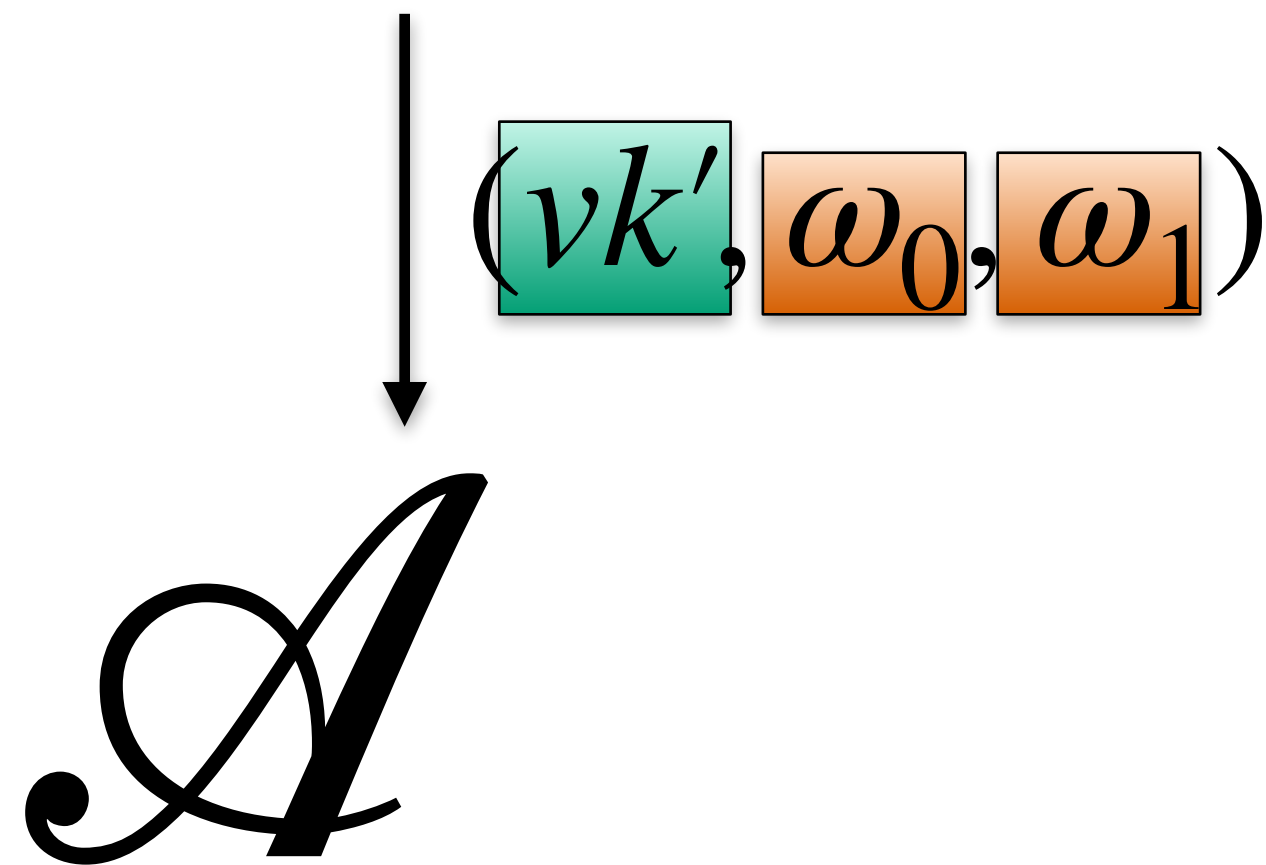
$$\begin{aligned} \boxed{vk'} &\leftarrow \text{ChangeRepPK}(\boxed{vk_b}, r) \\ \boxed{sk'} &\leftarrow \text{ChangeRepSK}(\boxed{sk_b}, r) \end{aligned} \quad r \leftarrow R$$

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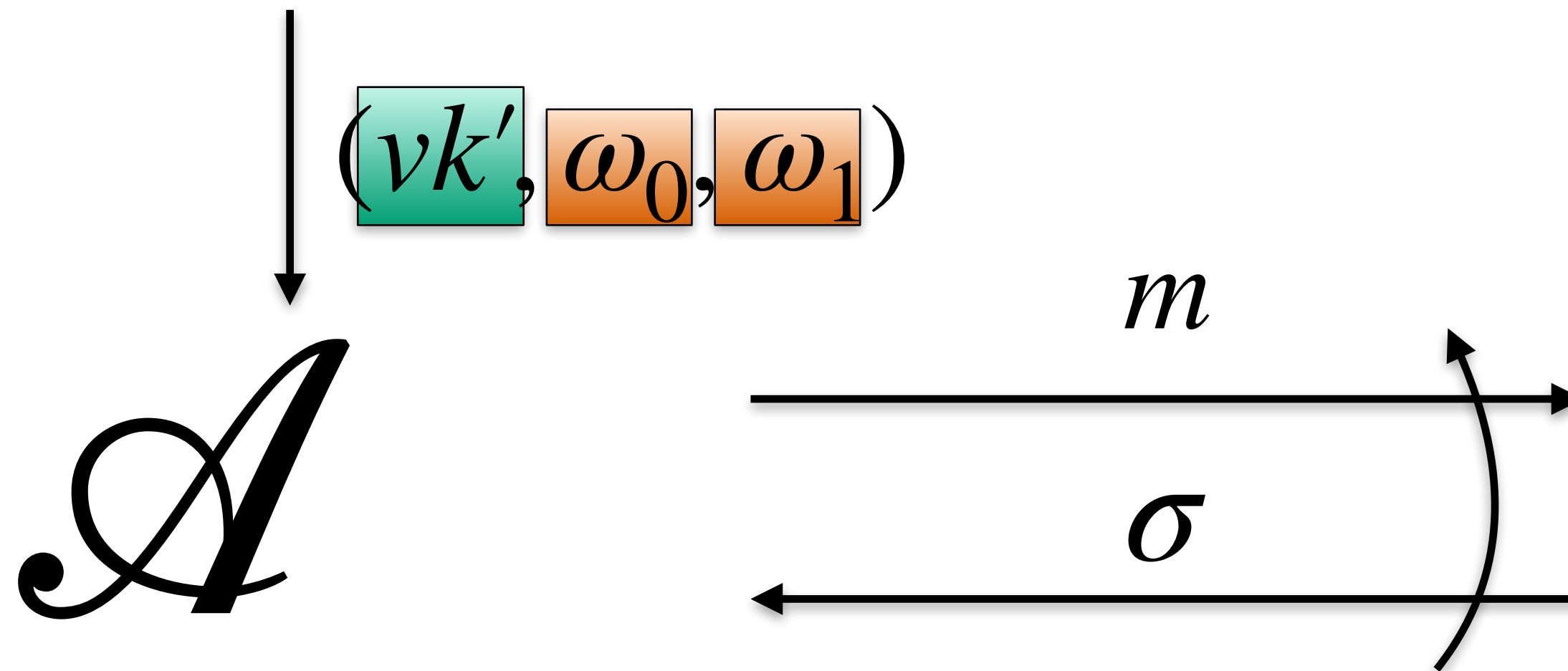
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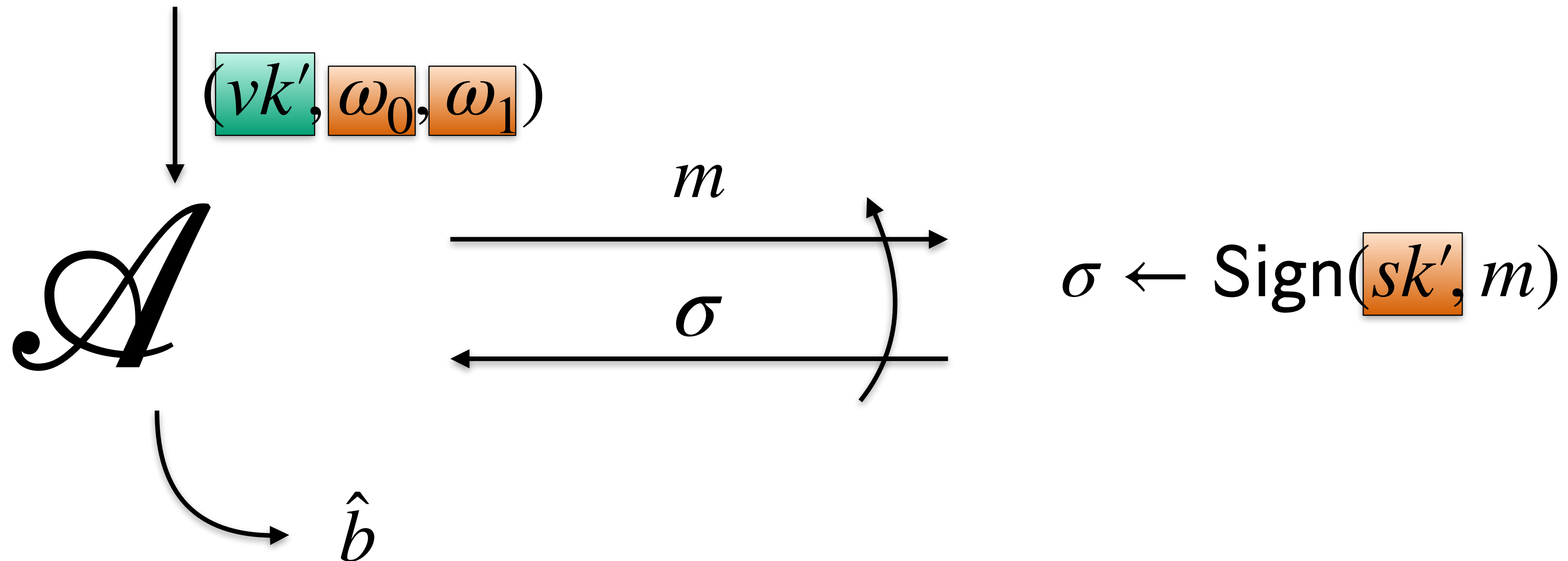


$$\sigma \leftarrow \text{Sign}(sk', m)$$

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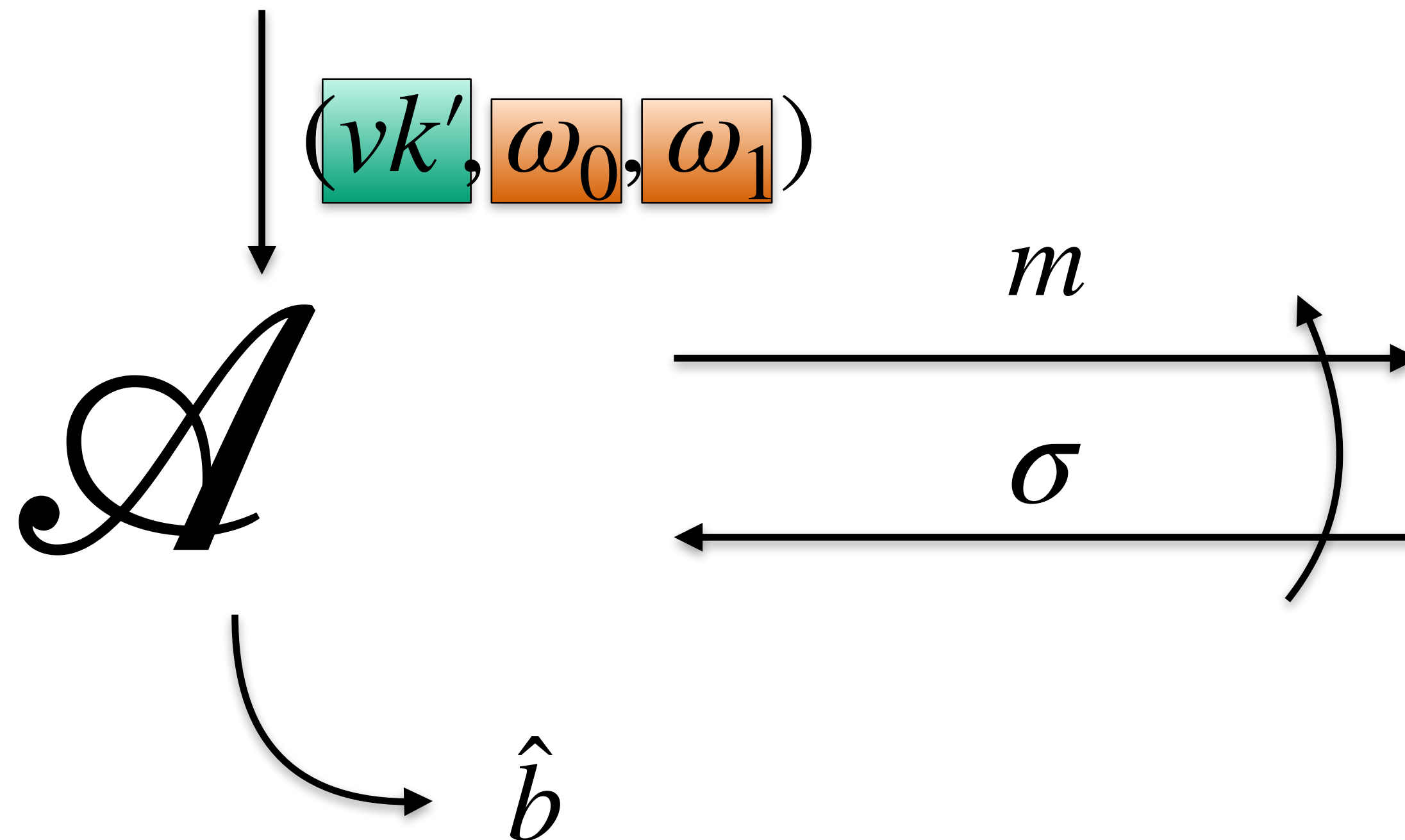
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$\mathcal{A}$ wins if

$$b = \hat{b}$$

$$\sigma \leftarrow \text{Sign}(sk', m)$$

$$\boxed{sk} = (y, X) \in \mathbb{Z}_p \times \mathbb{G}_1$$

$$\boxed{vk} = (A, B, C, D, t := e(X^y, g_2), K_{PHF}) \in \mathbb{Z}_p \times \mathbb{G}_1 \times \mathbb{G}_1^4 \times \mathbb{G}_T \times \mathbb{G}_1^{\lambda+1}$$

Sign

$$r \leftarrow \mathbb{Z}_p^*$$

$$\sigma_1 := \boxed{X^y} \cdot H_{\boxed{K_{PHF}}}(m)^r$$

$$\sigma_2 := g_1^r \quad \sigma_3 := g_2^r$$

Verify

$$e(\sigma_2, g_2) \stackrel{?}{=} e(g_1, \sigma_3)$$

$$e(\sigma_1, g_2) \stackrel{?}{=} e(\boxed{t}, g_2) \cdot e(H_{\boxed{K_{PHF}}}(m), \sigma_3)$$

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- Unforgeable under DLIN assumption and security of PHF (e.g. CDH)
- Class Hiding under DDH assumption in $\mathbb{G}_1$

Ring Signatures: [Malavolta-Schröder, 2017]

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- Security under q-Strong DH assumption + Linear-KEA (GGM)
- $O(n)$ signature size

Ring Signatures: Our Construction

$$sk' \leftarrow \Sigma . \text{ChangeRepSK}(sk, r)$$

$$\sigma_m \leftarrow \Sigma . \text{Sign}(sk', m || \mathcal{R})$$

Σ : Signature Scheme with Flexible Public Keys

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Σ : Signature Scheme with Flexible Public Keys

Π : perfectly sound NIWI [Groth-Ostrovsky-Sahai, 2006]

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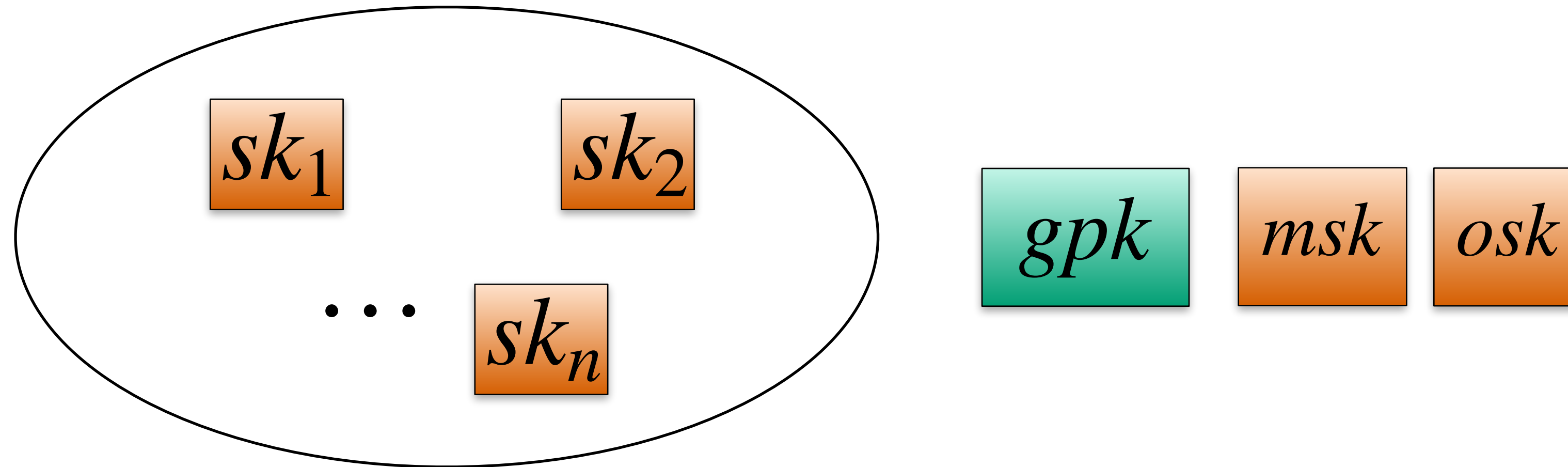
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Π : perfectly sound NIWI [Groth-Ostrovsky-Sahai, 2006]

- No need to extract randomizer from SFPK in unforgeability proof!
- Perfectly sound NIWI without Setup + square root technique [Chandran-Groth-Sahai, 2007]
➔ **$O(\sqrt{n})$ size Ring Signatures without Setup from Standard assumptions!**

Group Signatures [Bellare-Micciancio-Warinschi, 2003]

Group $\mathcal{G}$



$$\sigma \leftarrow \text{Sign}(gpk, sk_i, m)$$

$$\text{accept/reject} \leftarrow \text{Verify}(gpk, \sigma, m)$$

$$i \leftarrow \text{Open}(osk, \sigma, m)$$

Anonymity

- Signature hides signer identity

Traceability

- Opening cannot be evaded or subverted

Group Signatures: Our construction

Setup :

Sign :

Σ_{flex} : SFPK
 Σ_{EQ} : SPS-EQ
[Fuchsbauer-Gay, 2018]

}

$$vk, vk' \in \mathbb{G}^\ell$$

$$vk \sim vk' \text{ if there is } r \in \mathbb{Z}_p^*$$

$$(vk_1^r, \dots, vk_\ell^r) = (vk'_1, \dots, vk'_\ell)$$

Group Signatures: Our construction

Setup :

$$(\textcolor{brown}{msk}, \textcolor{teal}{gpk}[0]) \leftarrow \Sigma_{SPS} . \text{KeyGen}(1^\lambda)$$

$$\text{For } i \in [n] : (\textcolor{brown}{osk}[i], \textcolor{brown}{sk}_i, \textcolor{teal}{vk}_i) \leftarrow \Sigma_{flex} . \text{TKeyGen}(1^\lambda)$$

$$(\textcolor{teal}{gpk}[1], \dots, \textcolor{teal}{gpk}[n]) \leftarrow (\Sigma_{SPS} . \text{Sign}(\textcolor{brown}{msk}, \textcolor{teal}{vk}_1), \dots, \Sigma_{SPS} . \text{Sign}(\textcolor{brown}{msk}, \textcolor{teal}{vk}_n))$$

Sign :

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[Fuchsbauer-Gay, 2018]

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Sign :

$$sk' \leftarrow \Sigma_{flex} \cdot \text{ChangeRepSK}(sk, r)$$

$$\sigma_m \leftarrow \Sigma_{flex} \cdot \text{Sign}(sk', m)$$

$$\sigma'_{vk} \leftarrow \Sigma_{EQ} \cdot \text{ChangeRep}(gpk[0], vk', \sigma_{vk})$$

$$\sigma = (vk', \sigma_m, \sigma_{vk'})$$

Σ_{flex} : SFPK

Σ_{EQ} : SPS-EQ

[Fuchsbauer-Gay, 2018]

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Σ_{flex} : SFPK

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[Fuchsbauer-Gay, 2018]

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- DDH + co-Flex-DH $\Rightarrow 20\mathbb{G}_1 + 5\mathbb{G}_2$

- GGM + co-Flex-DH $\Rightarrow 9\mathbb{G}_1 + 2\mathbb{G}_2$

[Fuchsbauer-Hanser-Slamanig, 2014]

Signatures with flexible public keys:

- ➡ short ring signatures from weak assumptions without setup
- ➡ short group signatures because of compatibility with SPS-EQ
- ➡ generic privacy preserving building block

Thanks!

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jonas.schneider@cispa-helmholtz.de