Chair of Applied Cryptography

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FRIEDRICH-ALEXANDER UNIVERSITÄT ERLANGEN-NÜRNBERG

FACULTY OF ENGINEERING

Multi-Key Homomorphic Signatures Unforgeable under Insider Corruption

Russell W. F. Lai[1,2], Raymond K. H. Tai[2], Harry W. H. Wong[2], Sherman S. M. Chow[2]

[1]Friedrich-Alexander University Erlangen-Nuremberg

[2]Chinese University of Hong Kong

Useful multi-key homomorphic signatures likely require strong assumptions.

Overview

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We draw connections of the notion to zk-SNARG/Ks.

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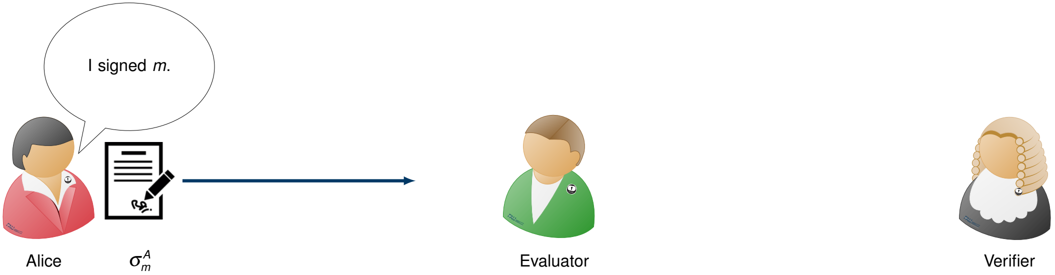

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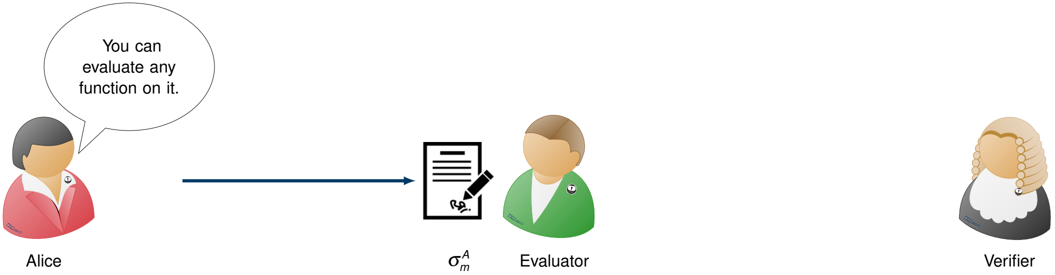

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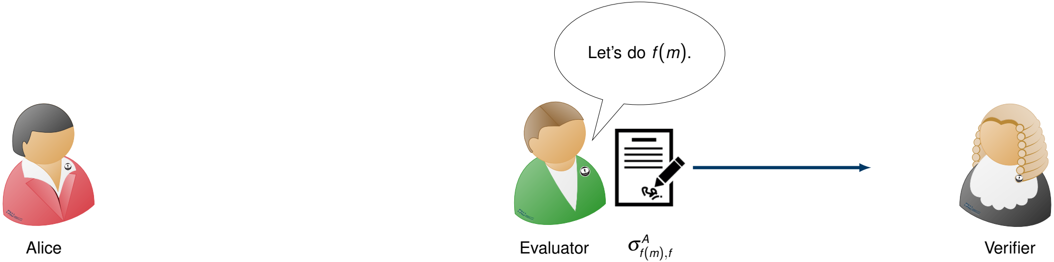

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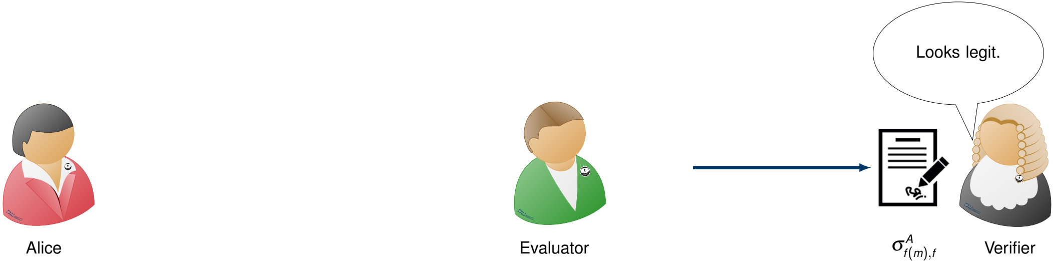

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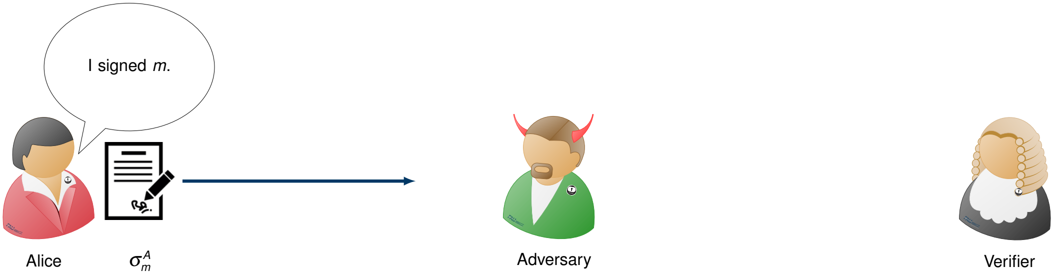

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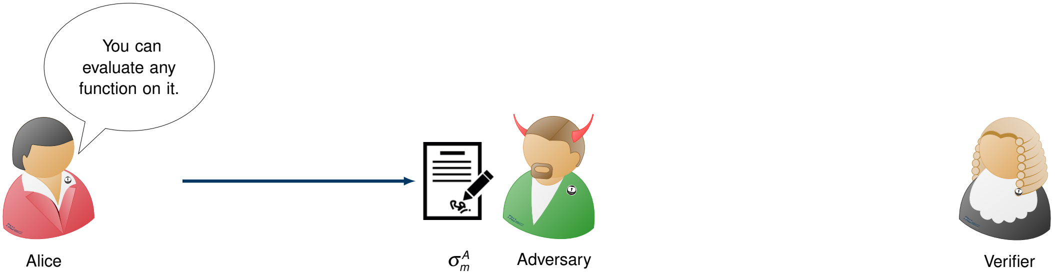

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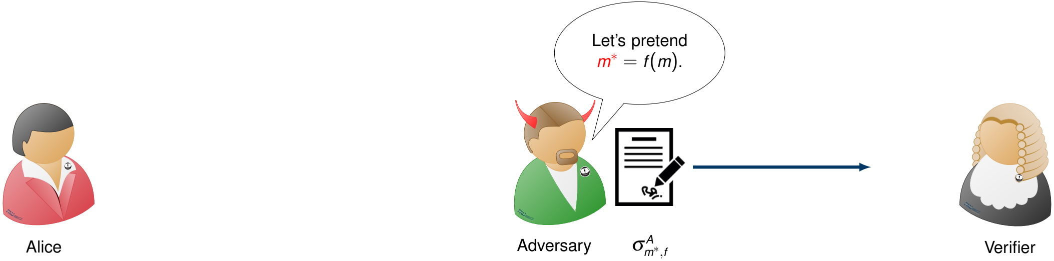

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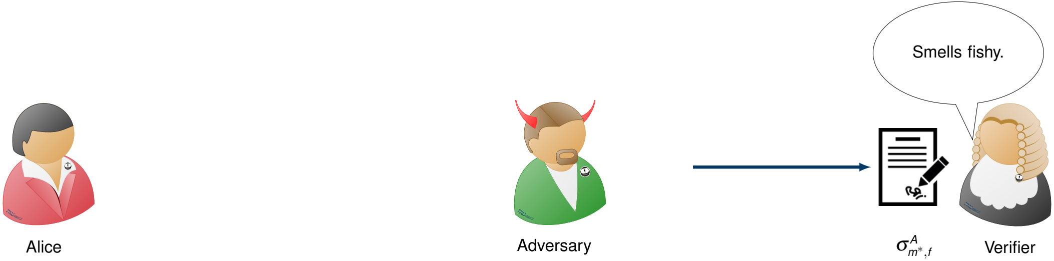

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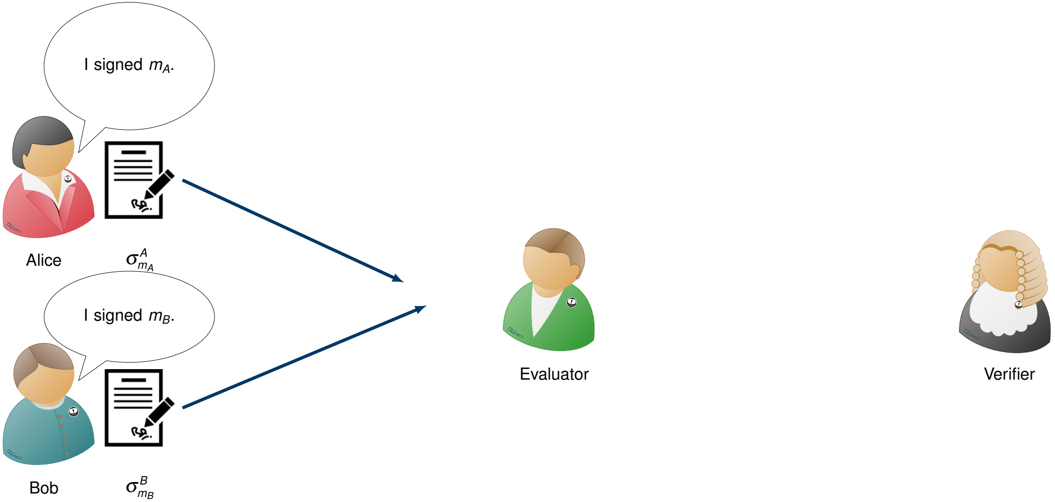

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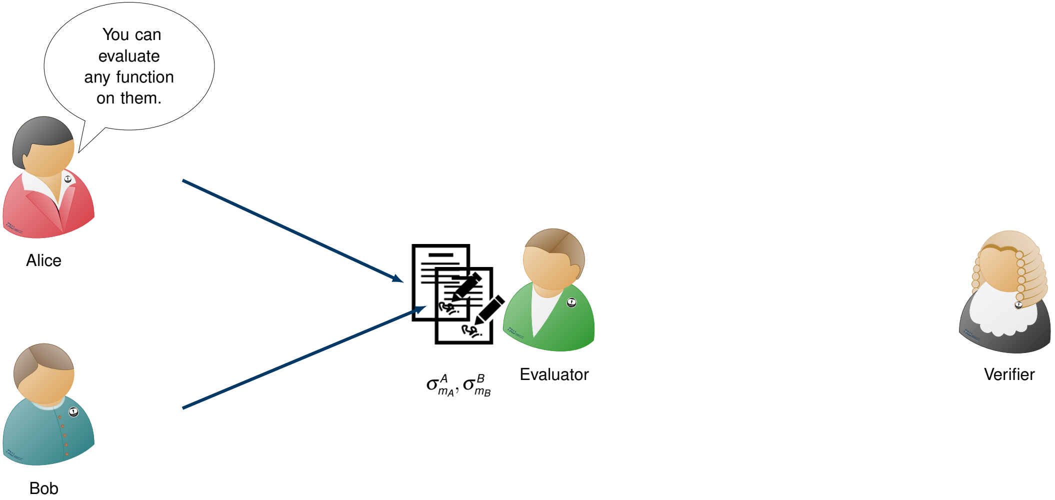

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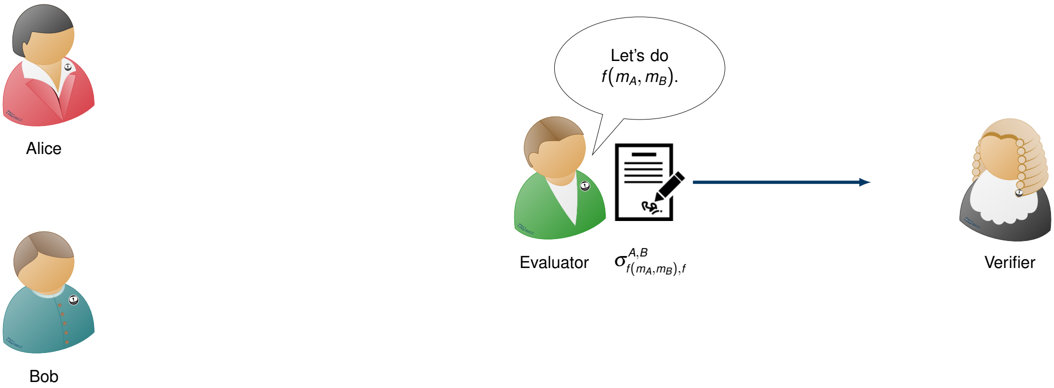

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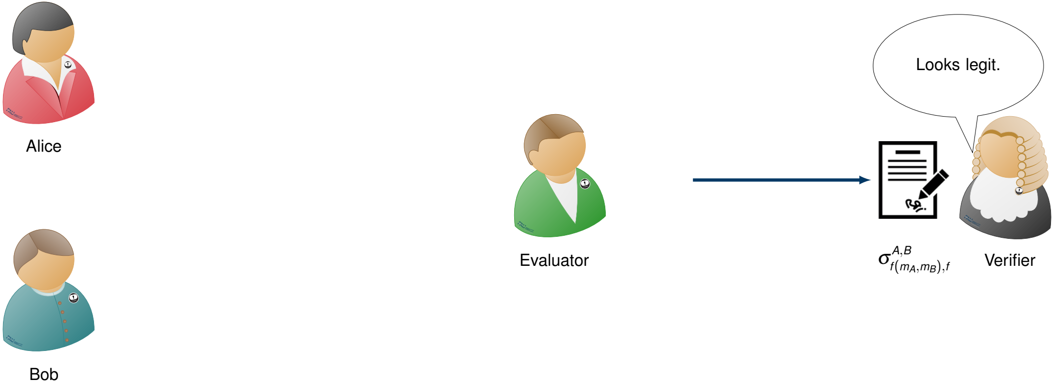

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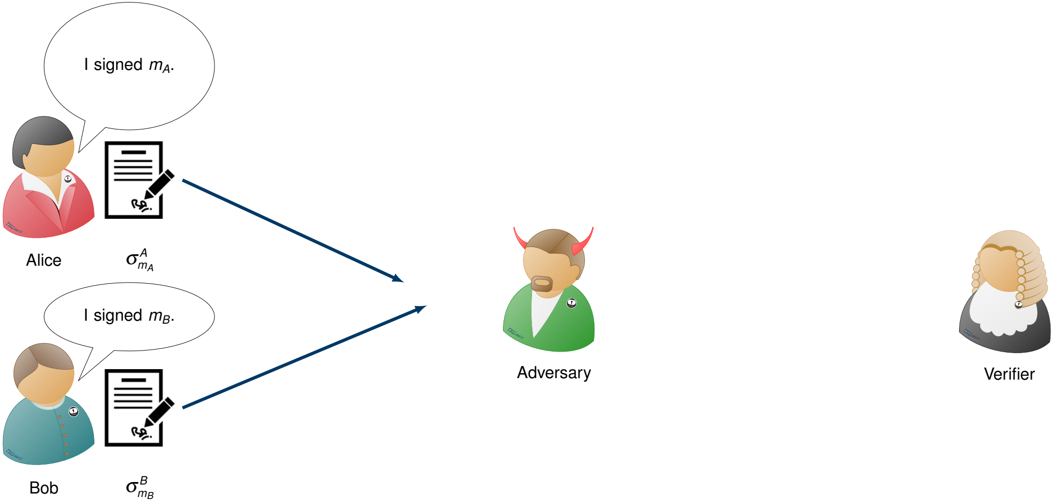

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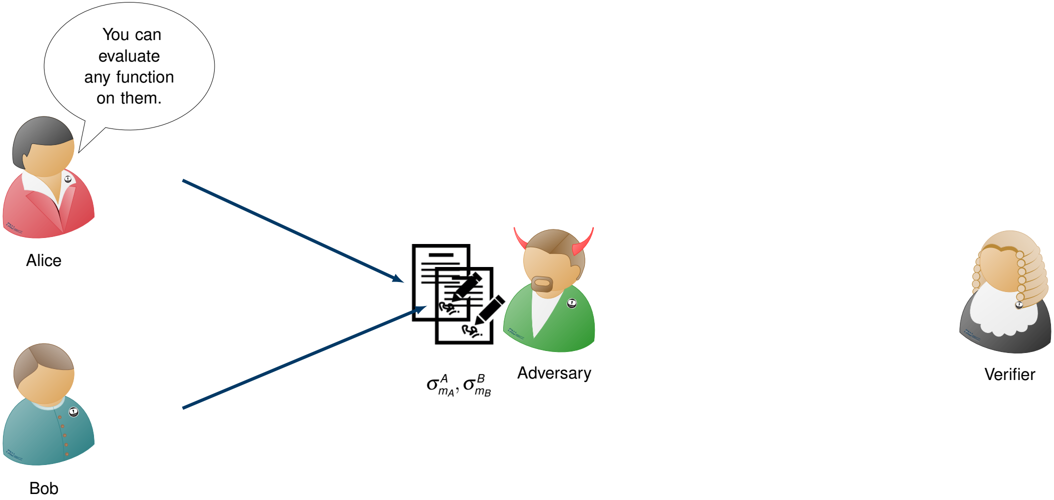

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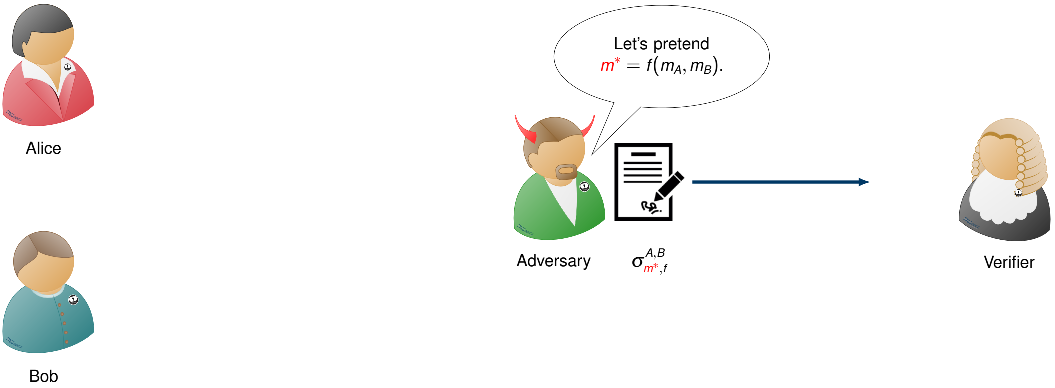

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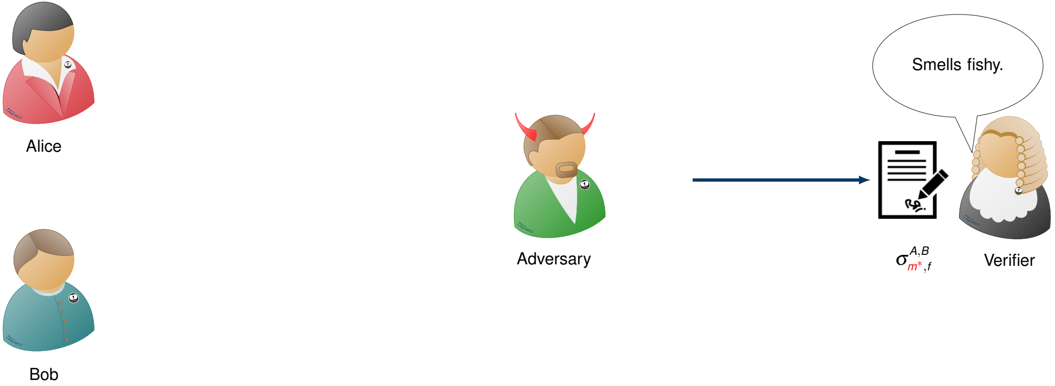

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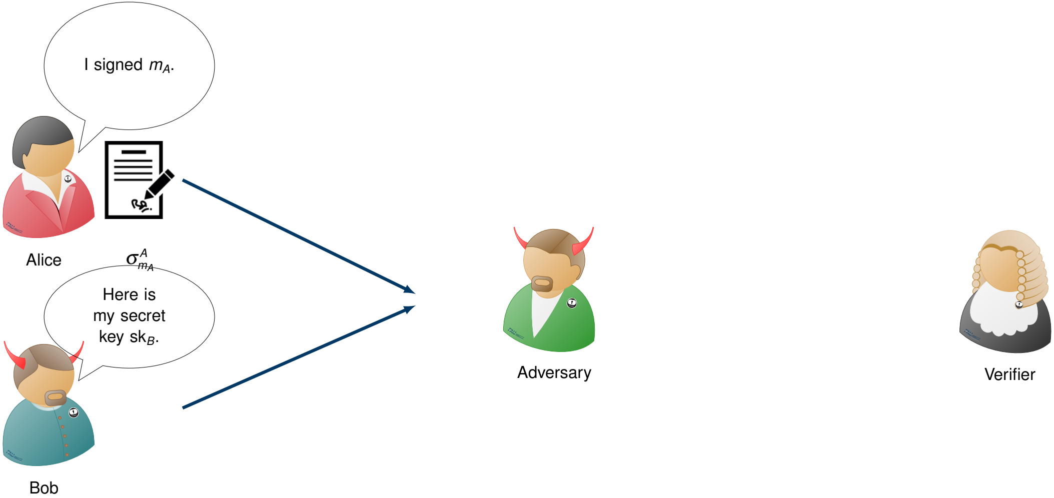

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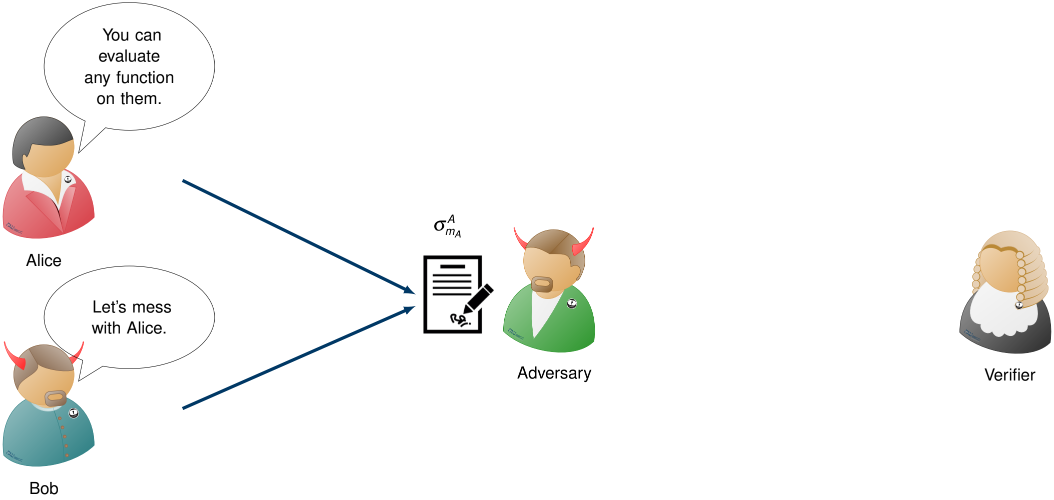

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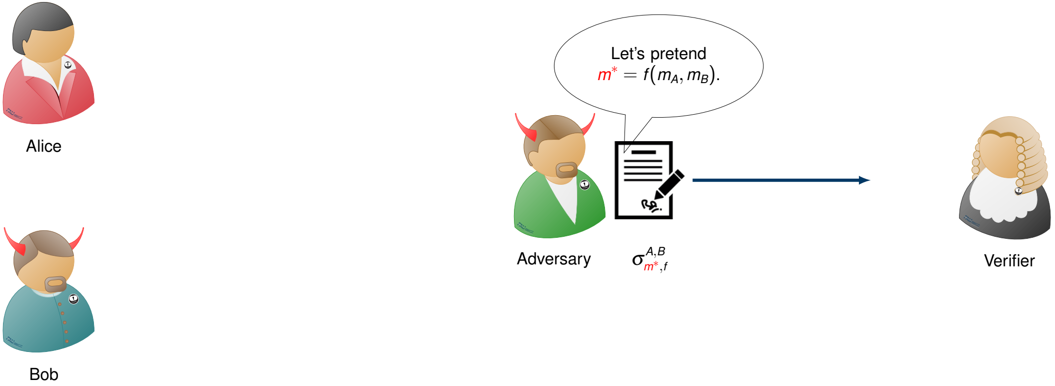

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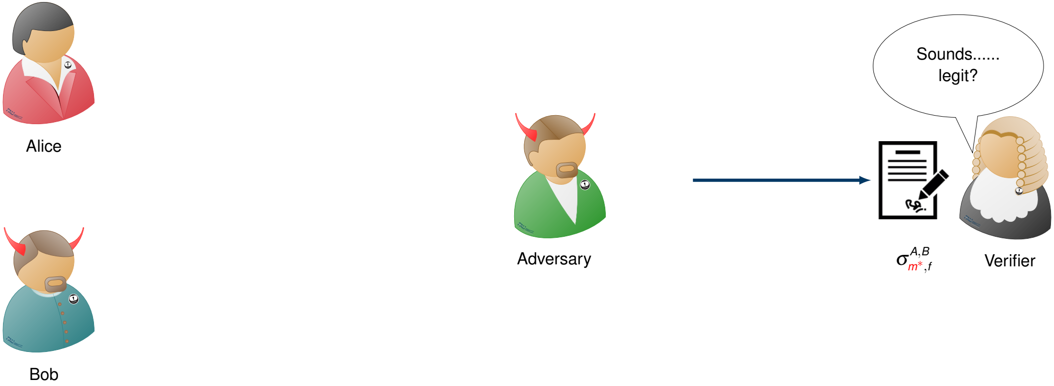

Unforgeability of (Multi-Key) Homomorphic Signatures under Insider Corruption

- $\mathcal{A}$ can query sign oracle on (id, m), which does the following:
  - Generate $(\mathsf{pk}_{\mathrm{id}}, \mathsf{sk}_{\mathrm{id}})$ and record id as honest if not done already.
  - Sign m using $\mathsf{sk}_{\mathrm{id}}$ as σ_m^{id} and record m in the set M_{id}.
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- $\mathcal{A}$ wins if the following hold:
  - $\mathsf{Vf}(f^*, \{\mathsf{pk}^*_{\mathsf{id}_1}, \ldots, \mathsf{pk}^*_{\mathsf{id}_k}\}, m^*, \sigma^*) = 1$.
  - If id is honest, then $\mathsf{pk}^*_{\mathsf{id}} = \mathsf{pk}_{\mathsf{id}}$.
  - m^* is not in the range of f^*, when the inputs of honest id are restricted to those recorded in M_{id},

    $$\text{i.e., } m^* \notin \left\{ f^*(m_1, \ldots, m_k) : \begin{cases} m_i \in \mathcal{M} & \mathsf{id}_i \text{ is malicious} \\ m_i \in M_{\mathsf{id}_i} & \mathsf{id}_i \text{ is honest} \end{cases} \right\}$$

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Remark

- The definition still makes sense even with one key, *i.e.*, $k = 1$.
- It means that even the signer cannot produce $\sigma_{m,f}$ for m not in the range of f.

Why is the notion meaningful?

Example 1: Number of keys $k > 1$

- $f^*(m_1, \ldots, m_k) = \mathsf{MAJORITY}(m_1, \ldots, m_k)$
- id_k malicious
- id_i honest, $\mathsf{M}_{\mathsf{id}_i} = \{\mathrm{NO}\}$, for all $i = 1, \ldots, k-1$
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Example 2: Number of keys $k = 1$

- C: Unsatisfiable Boolean circuit
- $f^*(m) = C(m)$
- Infeasible to forge $(C, \text{pk}, m^* = 1, \sigma^*)$

Other Properties of (Multi-key) Homomorphic Signatures

(Weakly) Context-Hiding $\sigma_{f(m),f}$ reveals nothing about m.

Succinctness Size of $\sigma_{f(m),f}$ is independent of the size of m and f.

Preliminary: zk-(O-)SNARG/Ks

Argument systems which allow a prover to prove to the verifier:

There exists a witness w such that the relation $R(x, w) = 1$ holds for the statement x.

zero-knowledge	: Proofs reveal nothing about witnesses.
Oracle	: Sound even if the prover has access to certain (*e.g.*, signing) oracles.
Succinct	: Proof size is independent of witness size.
Non-Interactive	: The prover only sends 1 message to the verifier.
ARGguments	: The system is computationally sound.
ARguments of Knowledge	: There exists an extractor which extracts witnesses from provers.

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Homomorphic signatures cannot be proven unforgeable under insider corruption via a black-box reduction from any falsifiable assumption (assuming messages can depend on public parameters).

Construction of Homomorphic Signatures

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- zk-(O-)SNARK Π
- Digital signature scheme Σ

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- Evaluation: Given g, $\{(f_i, \mathrm{pk}_i, m_i, \sigma^i_{m_i,f_i})\}_{i=1}^k$,
  - Let $h = g(f_1, \ldots, f_k)$.
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  - Produce a zk-SNARK proof for the following statement:

$$\text{“}I\ know\ g\ and\ \{(f_i, m_i, \sigma^i_{m_i,f_i})\}_{i=1}^k\ such\ that$$
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- Verification: If signature is fresh, use verification of Σ. If signature is evaluated, use verification of Π.

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  - ✗ There exists Σ $s.t.$ no candidate construction of O-SNARK satisfies proof of knowledge with respect to the signing oracle of Σ. [Fiore-Nitulescu, TCC16B]
  - ✓ Use a Σ which admits an O-SNARK. [Fiore-Nitulescu, TCC16B]

Construction of zk-SNARG

- Ingredients:
  - 1-key 1-hop homomorphic signature Σ unforgeable under insider corruption
  - A circuit g such that $g(x,w) = \begin{cases} x & R(x,w) = 1 \\ \perp & \text{otherwise} \end{cases}$.

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Soundness

If x^* is a NO instance, then $g(x^*, w) = \perp$ for all w.

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