Learning Strikes Again: the Case of the DRS Signature Scheme

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Asiacrypt 2018
Brisbane, Australia

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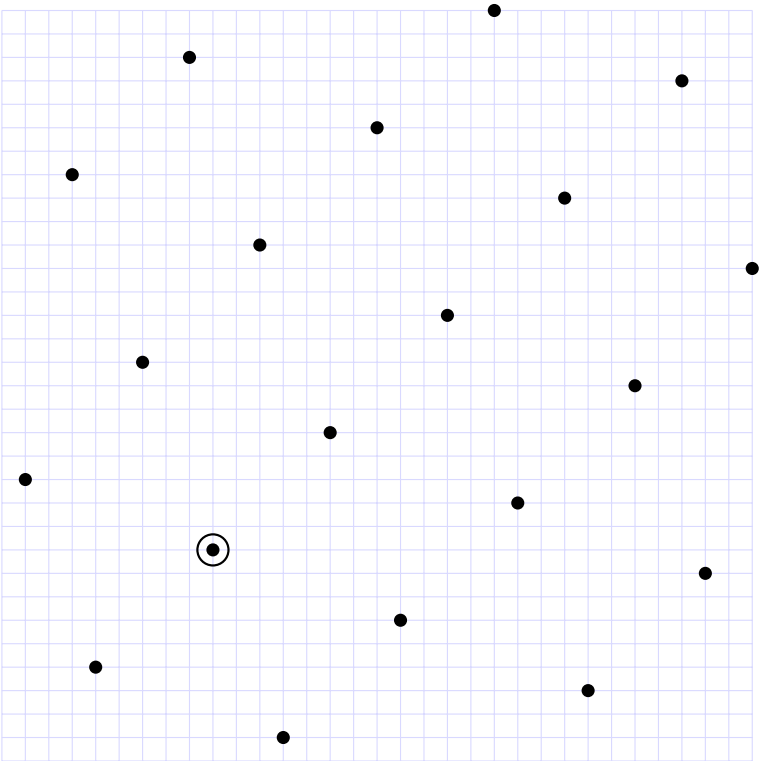
- Target: DRS — a NIST lattice-based signature proposal
- Techniques: learning & lattice
  - Statistical learning $\Rightarrow$ secret key information leaks
  - Lattice techniques $\Rightarrow$ better use of leaks
- They claim that Parameter Set-I offers **at least 128-bits** of security. We show that it actually offers **at most 80-bits** of security!

Outline

1. Background
2. DRS signature
3. Learning secret key coefficients
4. Exploiting the leaks

Outline

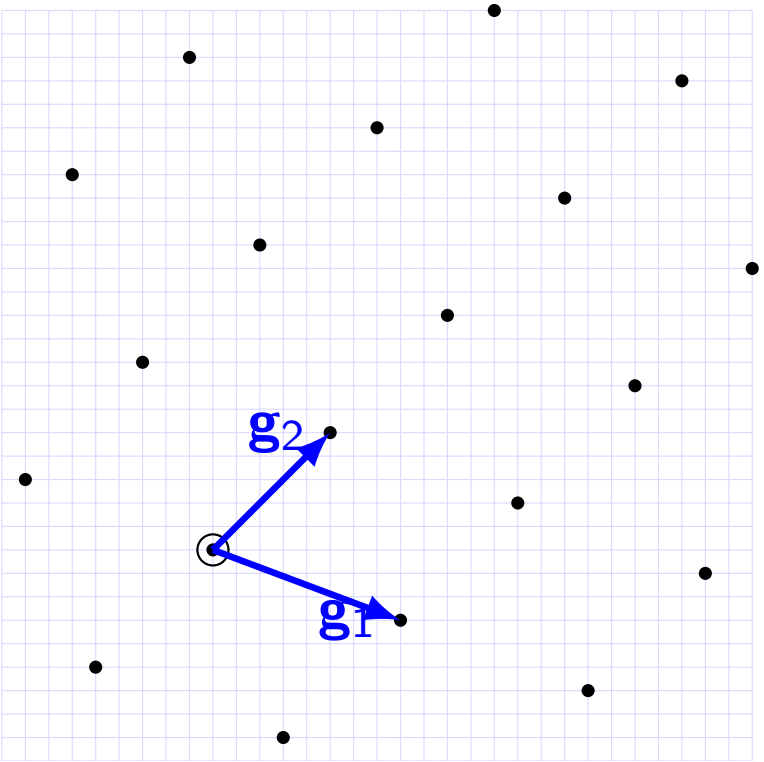
Lattice



Definition

A lattice $\mathcal{L}$ is a discrete subgroup of $\mathbb{R}^m$.

Lattice



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A lattice is generated by its basis $\mathbf{G} = (\mathbf{g}_1, \cdots, \mathbf{g}_n) \in \mathbb{R}^{n \times m}$, e.g. $\mathcal{L} = \{\mathbf{x}\mathbf{G} \mid \mathbf{x} \in \mathbb{Z}^n\}$.

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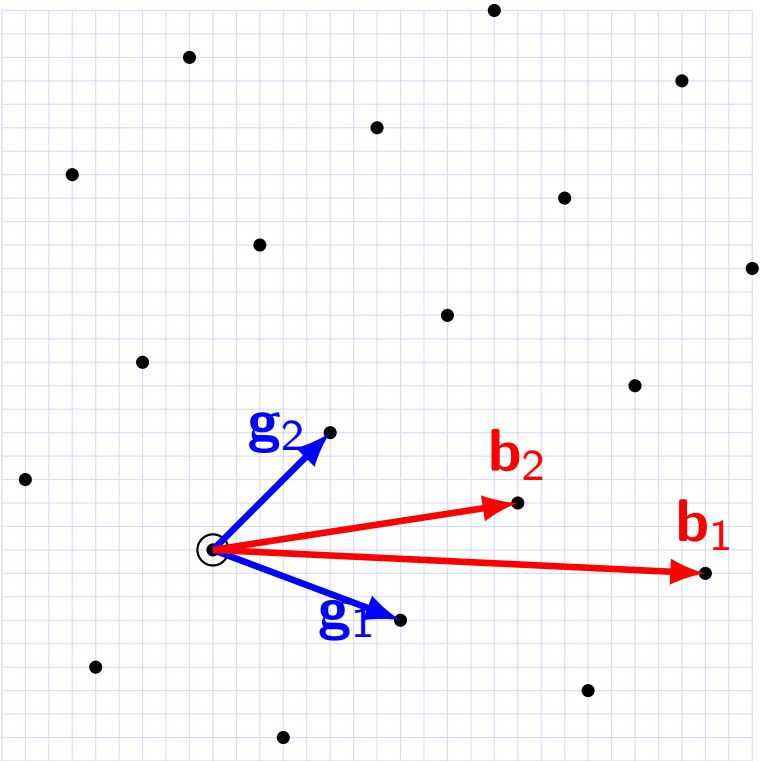

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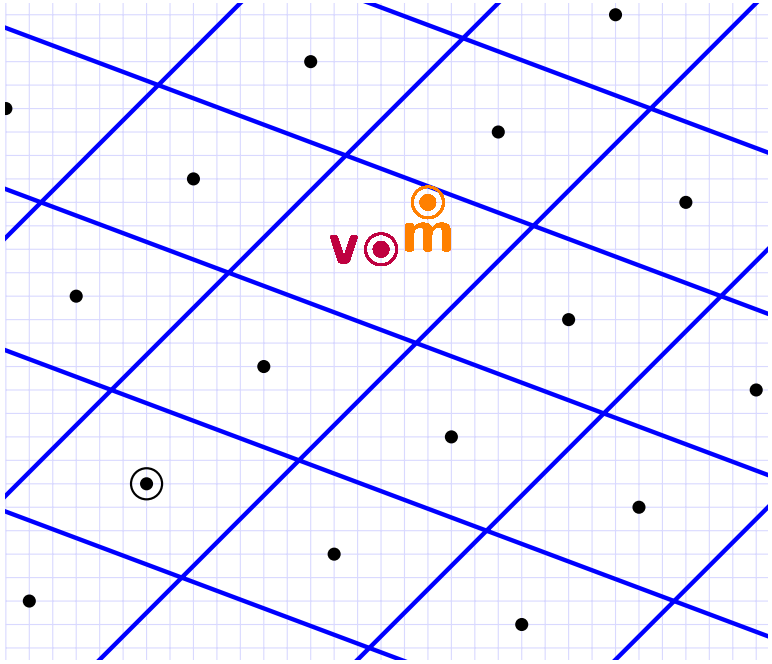
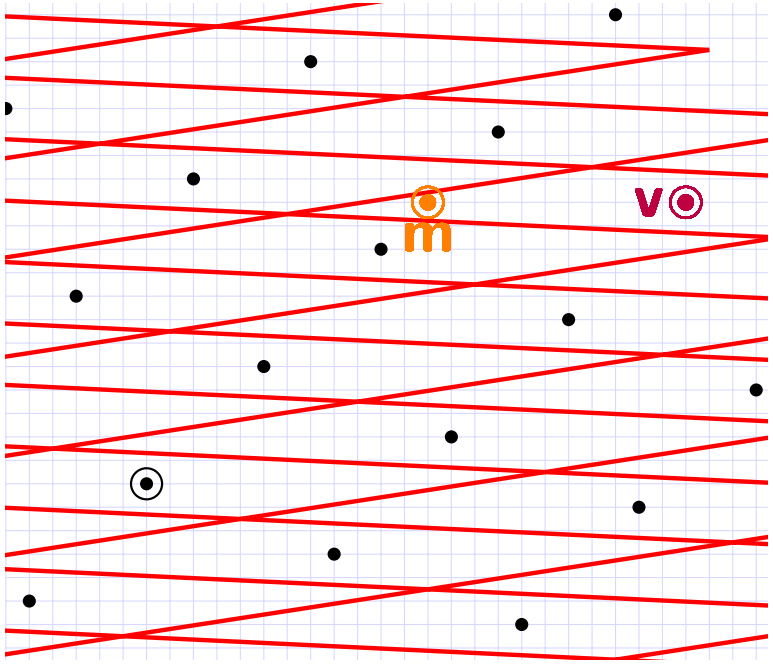
$\mathcal{L}$ has infinitely many bases
$\mathbf{G}$ is good, $\mathbf{B}$ is bad.

Finding Close Vectors

Each basis defines a parallelepiped $\mathcal{P}$.





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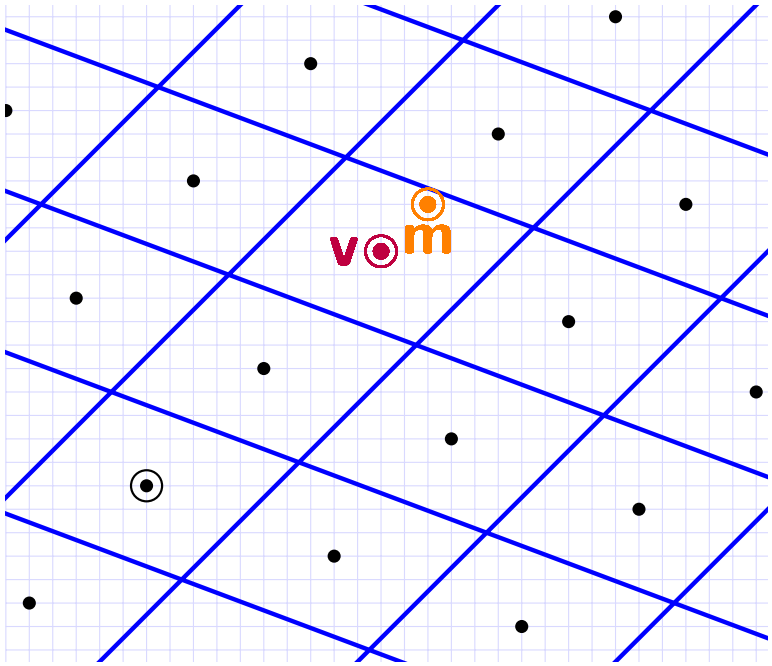
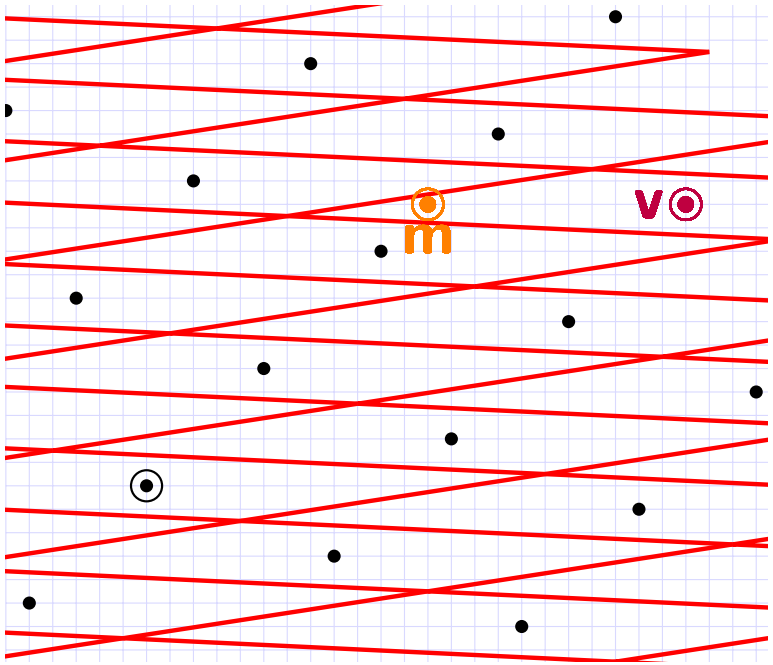




Babai's round-off algorithm outputs $\mathbf{v} \in \mathcal{L}$ such that $\mathbf{v} - \mathbf{m} \in \mathcal{P}$.

GGH & NTRUSign Schemes

Public key: $\mathbf{P}$, secret key: $\mathbf{S}$

Sign

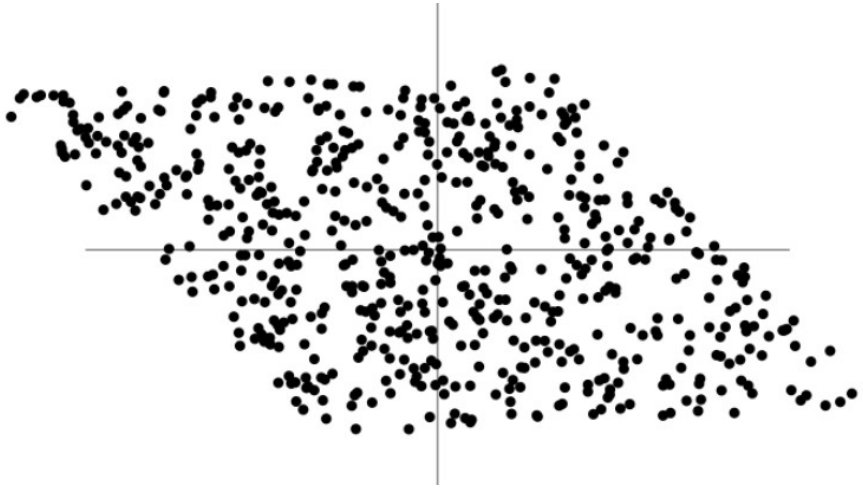
1. Hash the message to a random vector $\mathbf{m}$
2. Round $\mathbf{m}$ (using $\mathbf{S}$) to $\mathbf{v} \in \mathcal{L}$

Verify

1. Check $\mathbf{v} \in \mathcal{L}$ (using $\mathbf{P}$)
2. Check $\mathbf{v}$ is close to $\mathbf{m}$

GGH & NTRUSign are insecure!

$\mathbf{v} - \mathbf{m} \in \mathcal{P}(\mathbf{S}) \Rightarrow (\mathbf{v}, \mathbf{m})$ leaks some information of $\mathbf{S}$.



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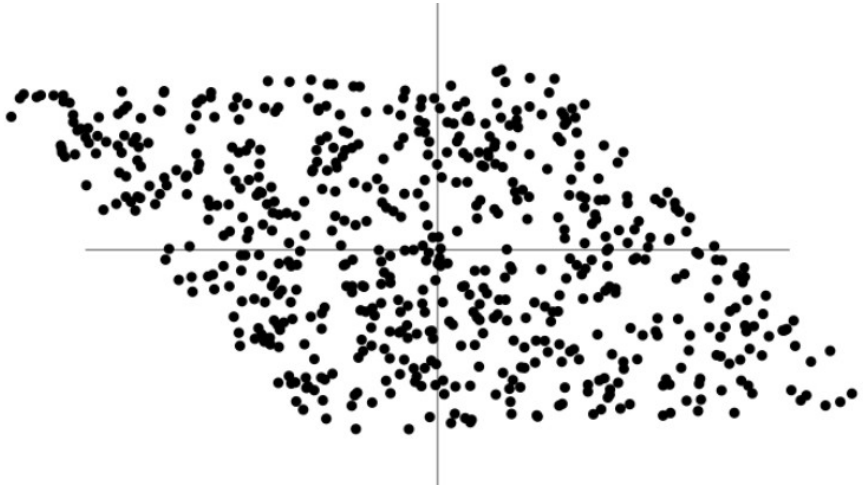

GGH and NTRUSign were broken by "learning the parallelepiped" [NR06].

Some countermeasures were also broken by a similar attack [DN12].

Countermeasures

Let us focus on Hash-then-Sign approach!

Provably secure method [GPV08]

- rounding based on Gaussian sampling
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Heuristic method [PSW08]

- rounding based on CVP w.r.t ℓ_∞-norm
- the support of $\mathbf{v} - \mathbf{m}$ is independent of $\mathbf{S}$
- DRS [PSDS17] is an instantiation, submitted to the NIST.

Outline

DRS

DRS = **D**iagonal-dominant **R**eduction **S**ignature

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Parameters: (n, D, b, N_b, N_1)

- n : the dimension
- D : the diagonal coefficient
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$$\mathbf{S} = \begin{pmatrix} D & & & \\ & D & & \\ & & \ddots & \\ & & & D \end{pmatrix} +$$

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$\leftarrow$ "absolute circulant"

Message reduction algorithm

Input: a message $\mathbf{m} \in \mathbb{Z}^n$, the secret matrix $\mathbf{S}$

Output: a reduced message $\mathbf{w}$ such that $\mathbf{w} - \mathbf{m} \in \mathcal{L}$ and $\|\mathbf{w}\|_\infty < D$

1: $\mathbf{w} \leftarrow \mathbf{m}, i \leftarrow 0$
2: **repeat**
3: $\quad \mathbf{w} \leftarrow \mathbf{w} - \lfloor \frac{w_i}{D} \rfloor \cdot \mathbf{s}_i$
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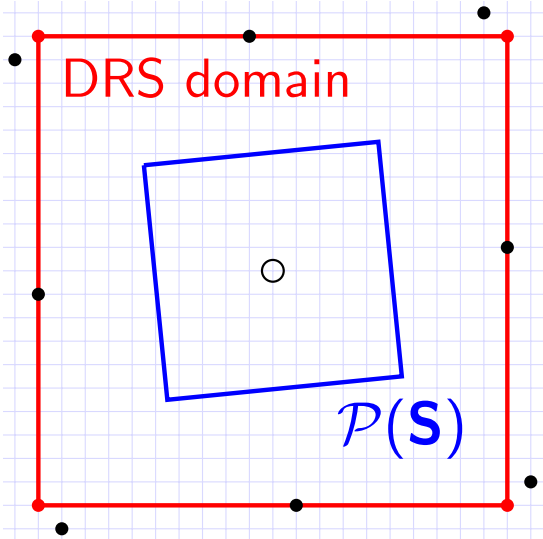
Intuition: use $\mathbf{s}_i$ to reduce

- w_i decreases a lot
- for $j \neq i$, w_j increases a bit
- $\|\mathbf{w}\|_1$ is reduced $\Rightarrow$ reduction always terminates!

Resistance to NR attack

The support of $\mathbf{w}$: $(-D, D)^n$



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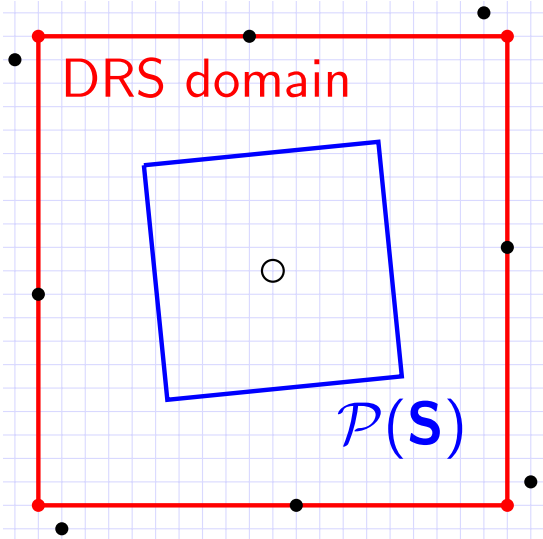

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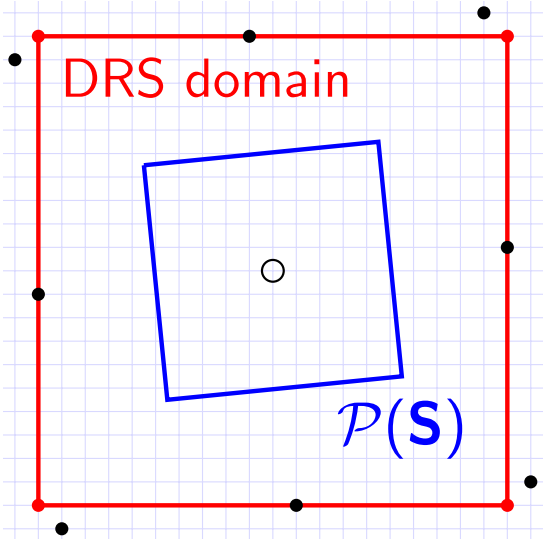

The support is "zero-knowledge", but **maybe the distribution is not!**

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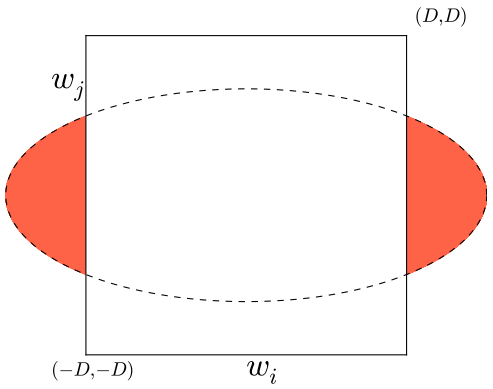
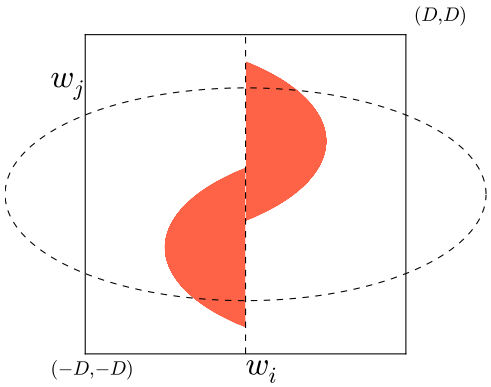
Intuition





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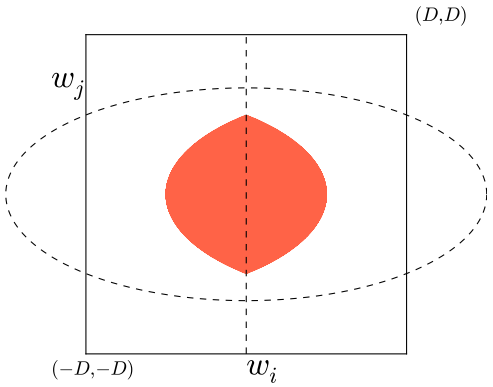

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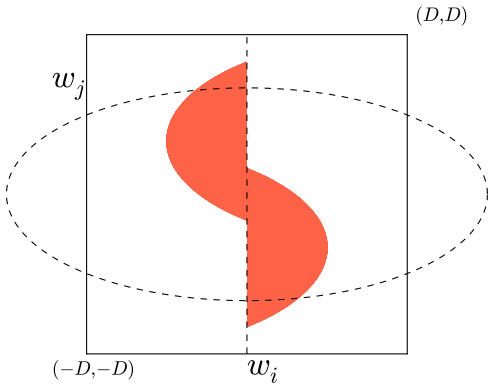

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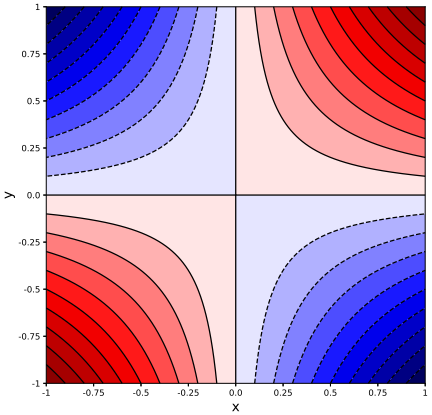
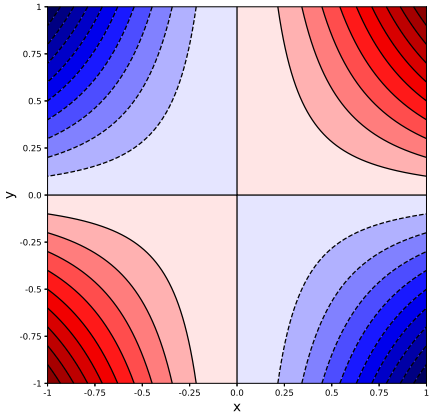
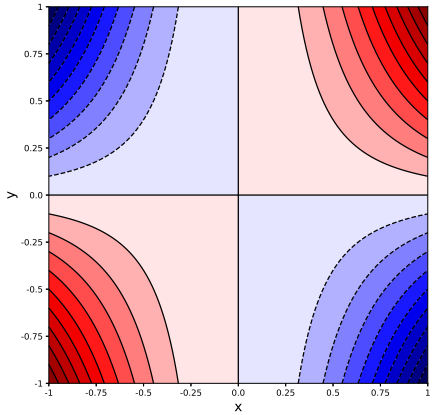
$\Rightarrow$ **Search for the best linear fit f ?**

Search space for all linear f: too large!

$\Rightarrow$ **choose some features $\{f_i\}$ and search in** $\mathrm{span}(\{f_i\})$**, i.e.** $f = \sum x_\ell f_\ell$







Training — feature selection

Lower degree moments:

$f_1(W) = \mathbb{E}(w_i w_j)$ $\quad$ $f_2(W) = \mathbb{E}(w_i \cdot |w_i|^{1/2} \cdot w_j)$ $\quad$ $f_3(W) = \mathbb{E}(w_i \cdot |w_i| \cdot w_j)$

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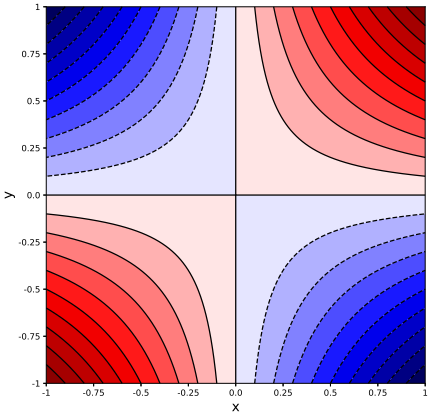

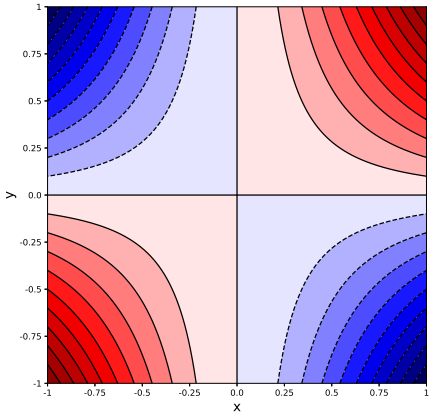

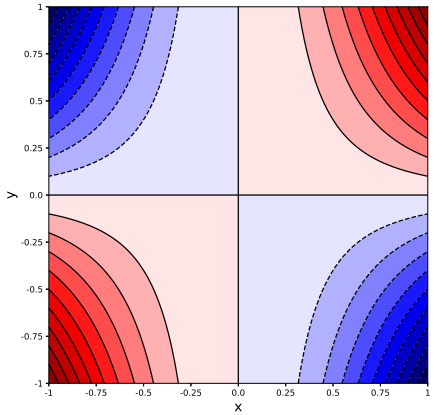

Not enough!

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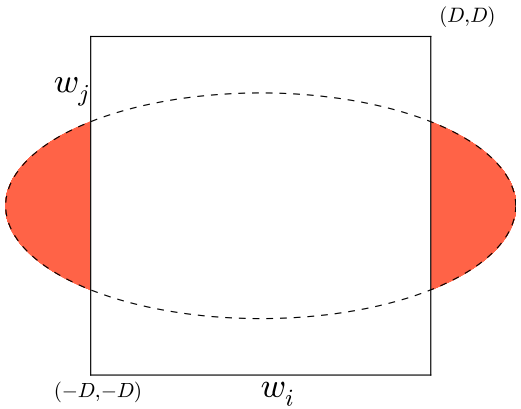

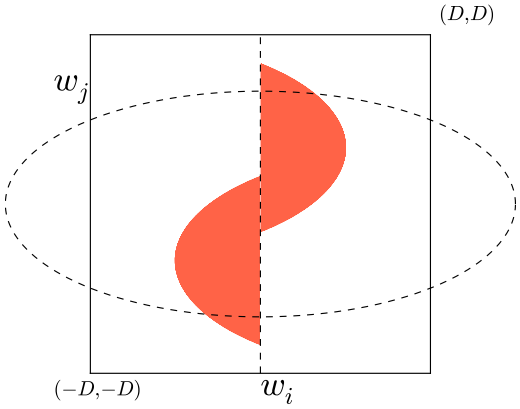

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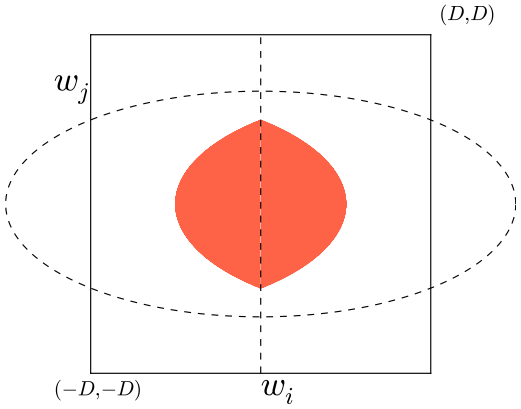

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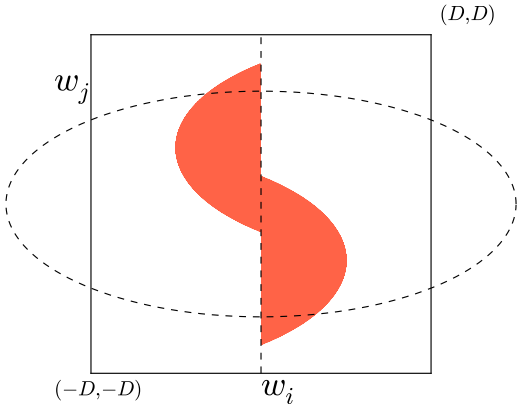
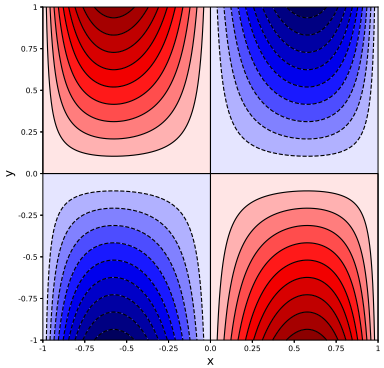


$\mathbf{S}_{i,j} = b$

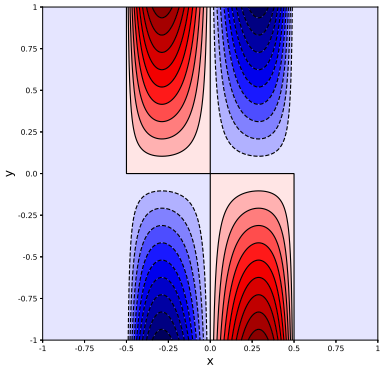
Training — feature selection

Pay more attention to the central region (i.e. $|w_i|$ small).

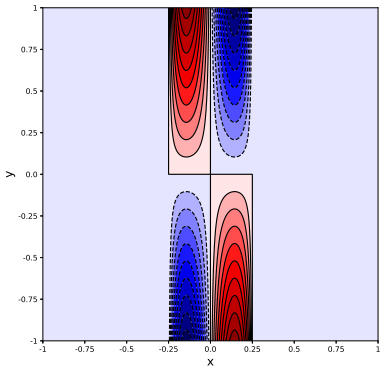
$$f_4 = \mathbb{E}(w_i(w_i - 1)(w_i + 1)w_j)$$



$$f_5 = \mathbb{E}(2w_i(2w_i - 1)(2w_i + 1)w_j \mid |2w_i| \leq 1)$$



$$f_6 = \mathbb{E}(4w_i(4w_i - 1)(4w_i + 1)w_j \mid |4w_i| \leq 1)$$



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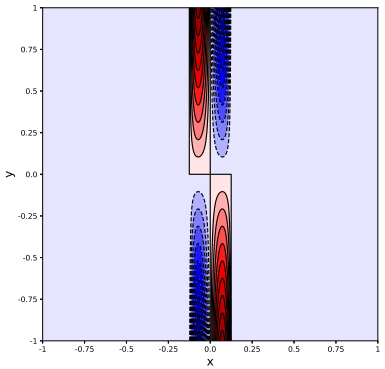

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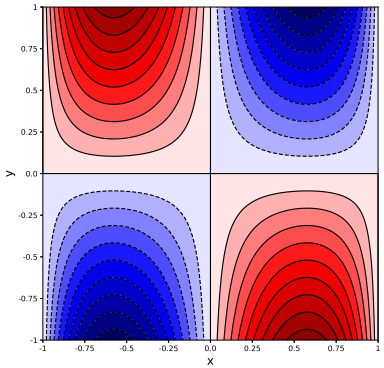

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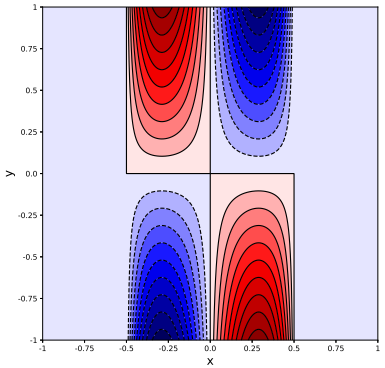

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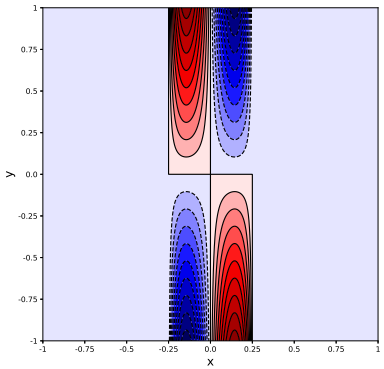

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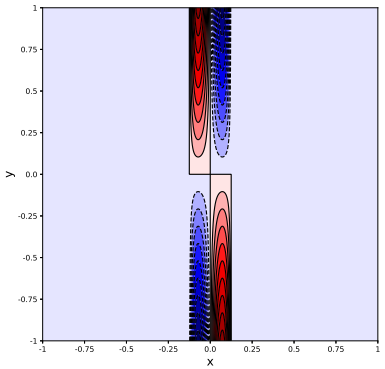
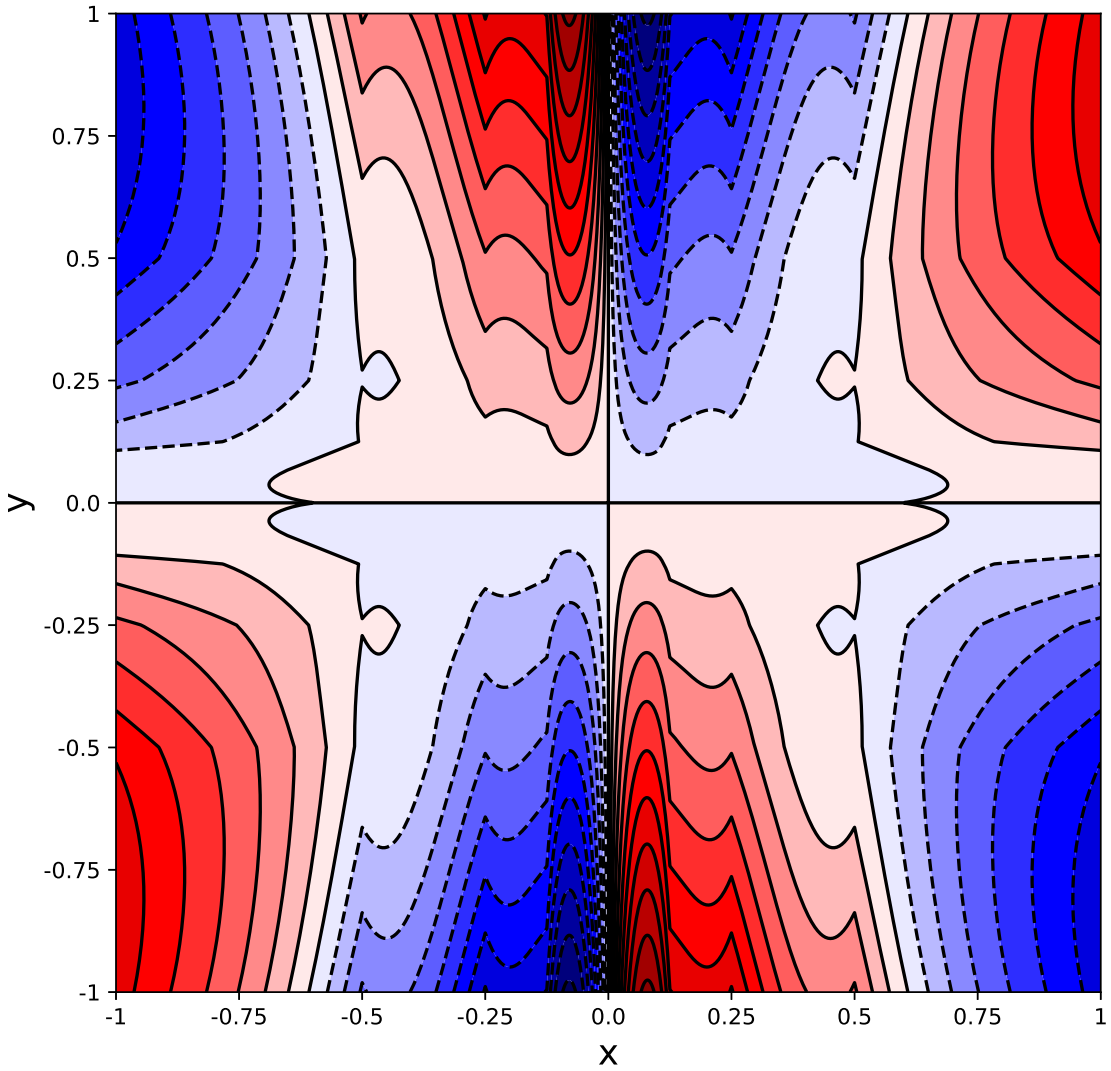


Together with transposes (i.e. $f^t(w_i, w_j) = f(w_j, w_i)$), we finally selected $7 \times 2 - 1 = 13$ features in experiments.

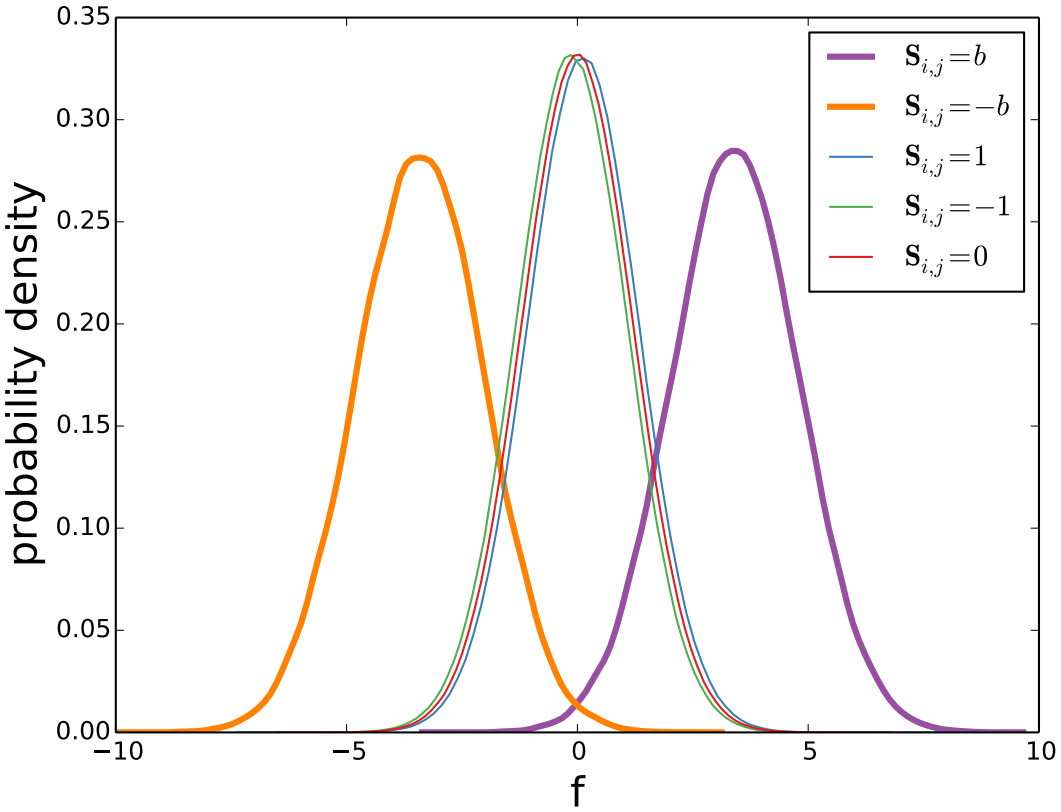
The model

$$f = \sum x_\ell f_\ell$$



Learning

Let's learn a new **S** as $\mathbf{S}' = f(W)$!



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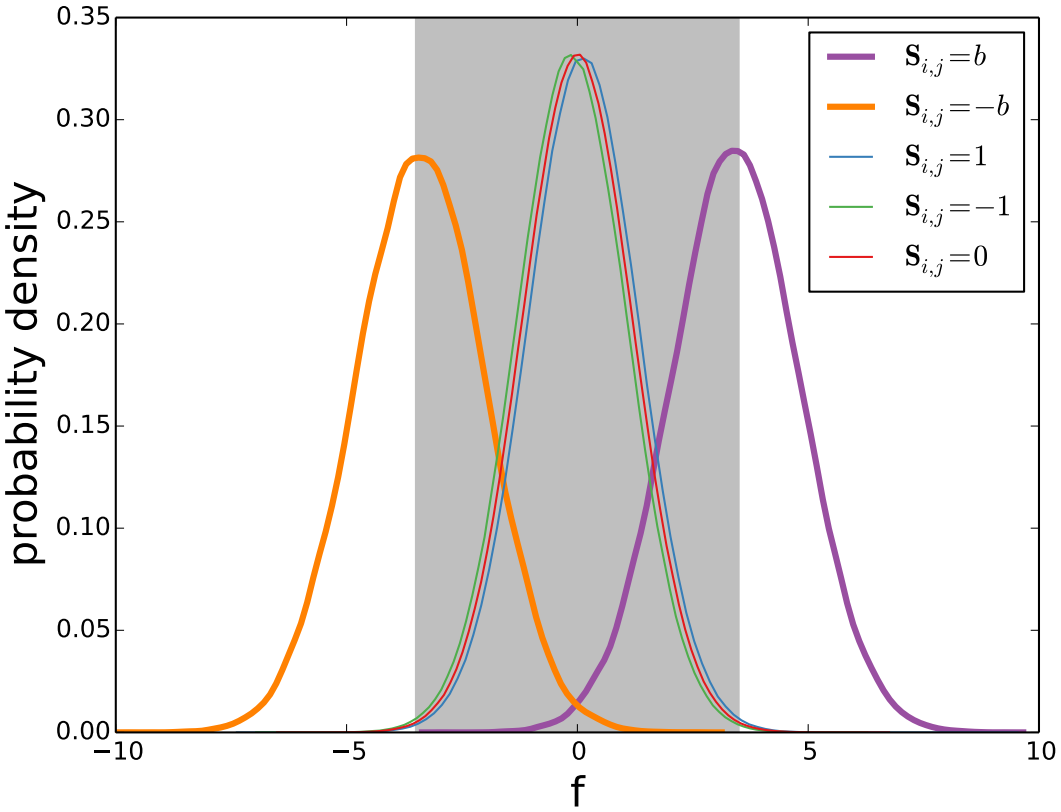
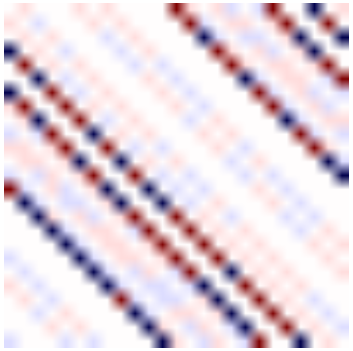
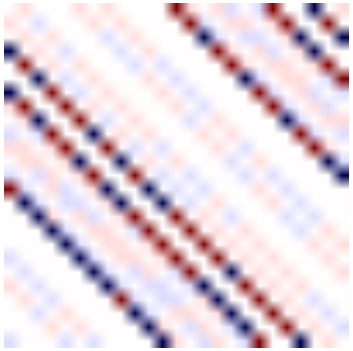


Learning

$\mathbf{S} = D \cdot I +$  is "absolute circulant"

$\Rightarrow$ more confidence via diagonal amplification

Learning

$\mathbf{S} = D \cdot I +$  is "absolute circulant"

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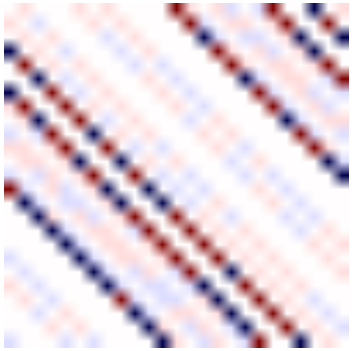
$\mathbf{S} = D \cdot I +$  is "absolute circulant"

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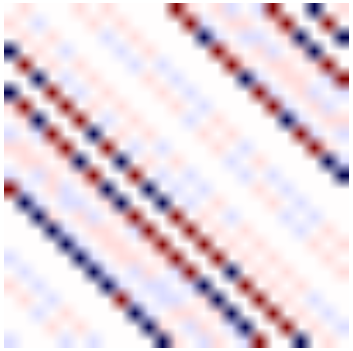
$\mathbf{S} = D \cdot I +$  is "absolute circulant"

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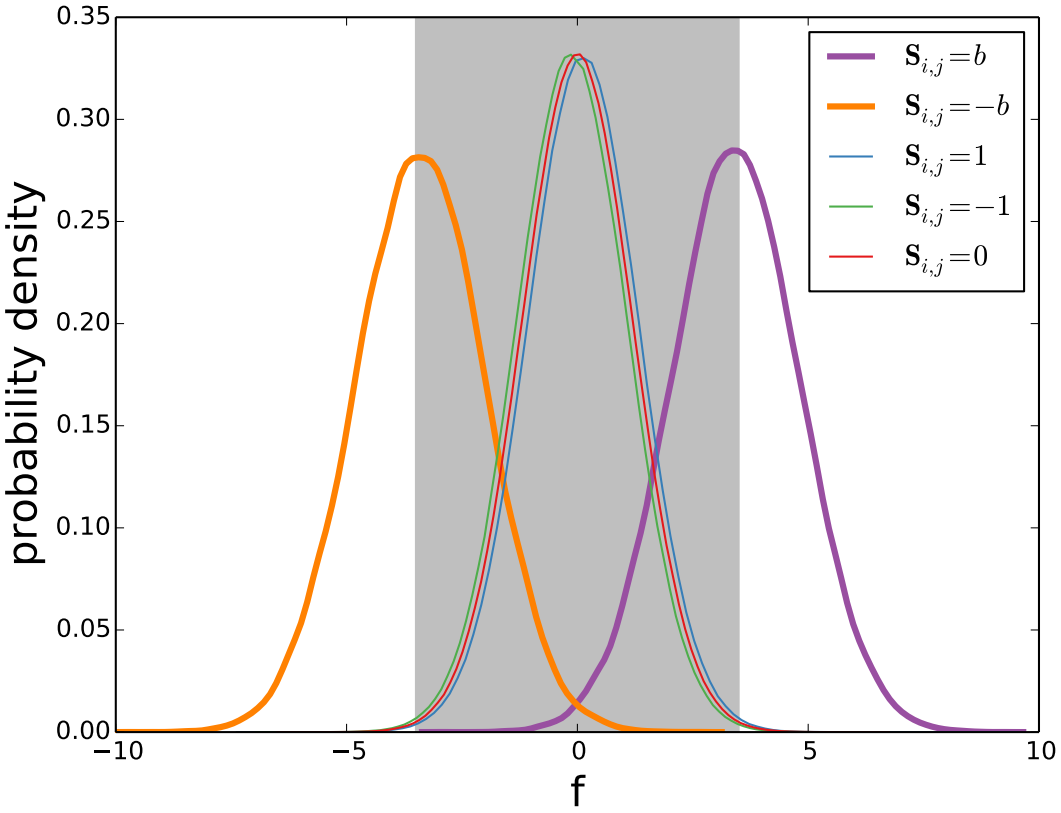
We locate all large coefficients successfully!

but we are still missing the signs!

Learning

$\mathbf{S}_{i,j} \in \{\pm b, \pm 1, 0\}$



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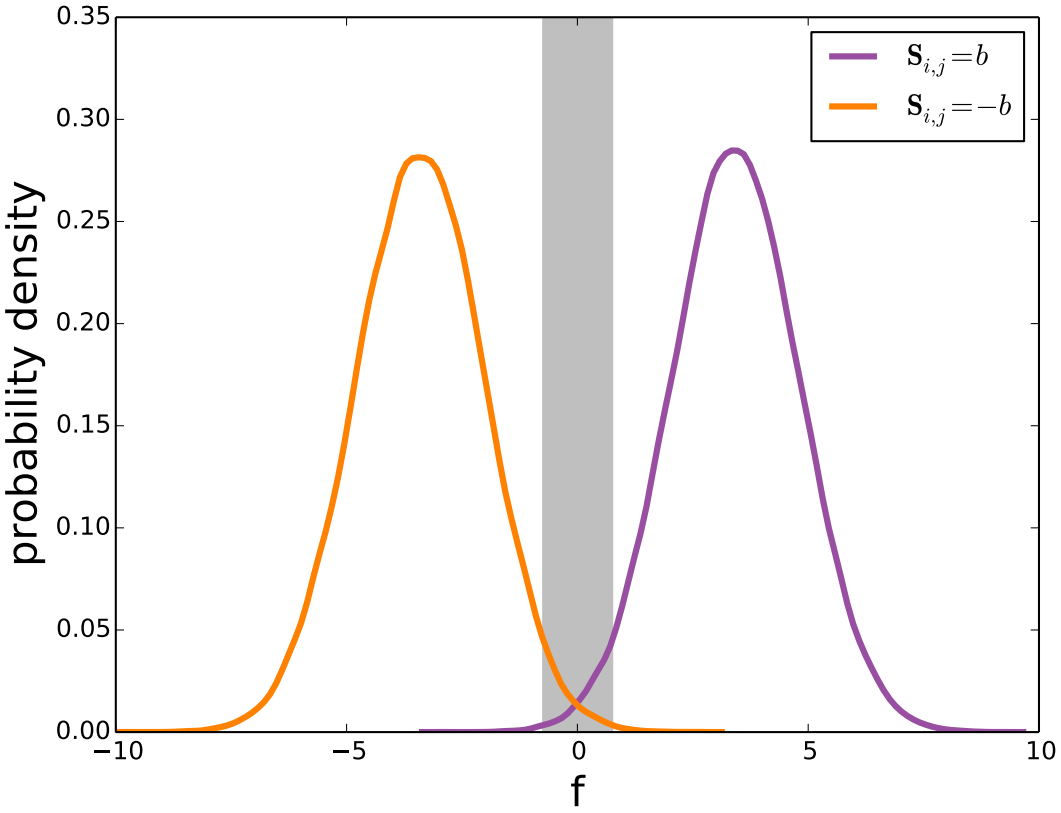

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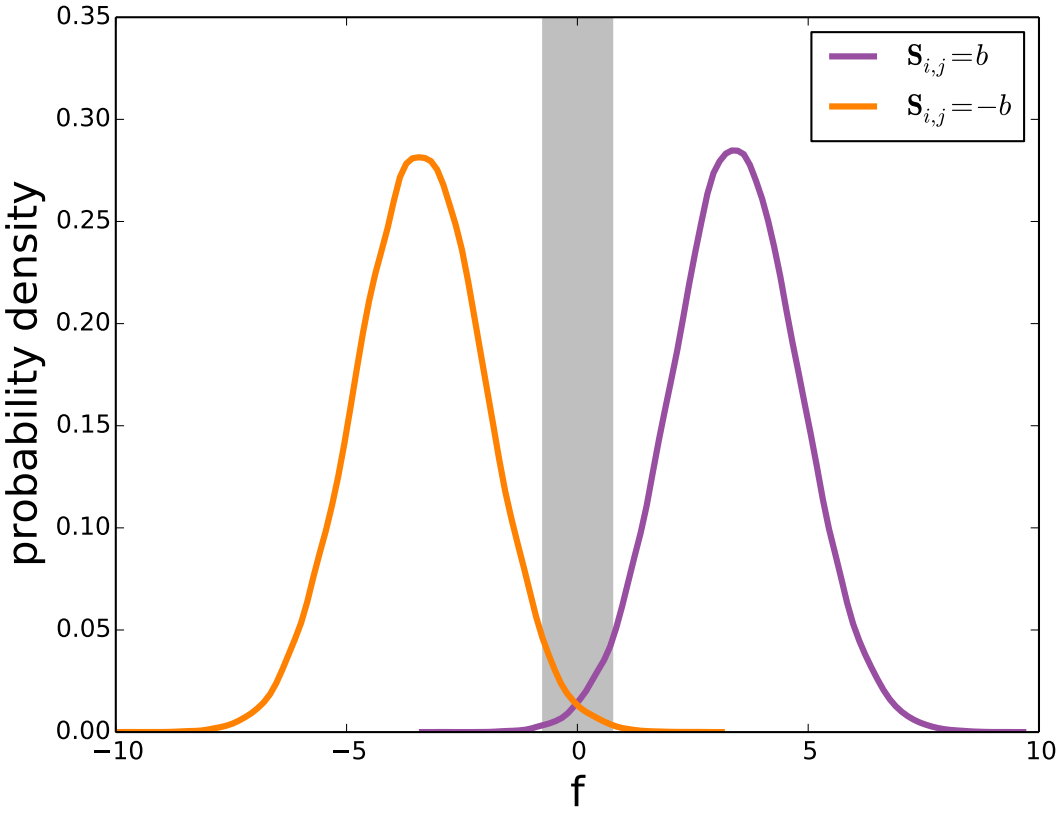

We can determine all large coefficients in one row!

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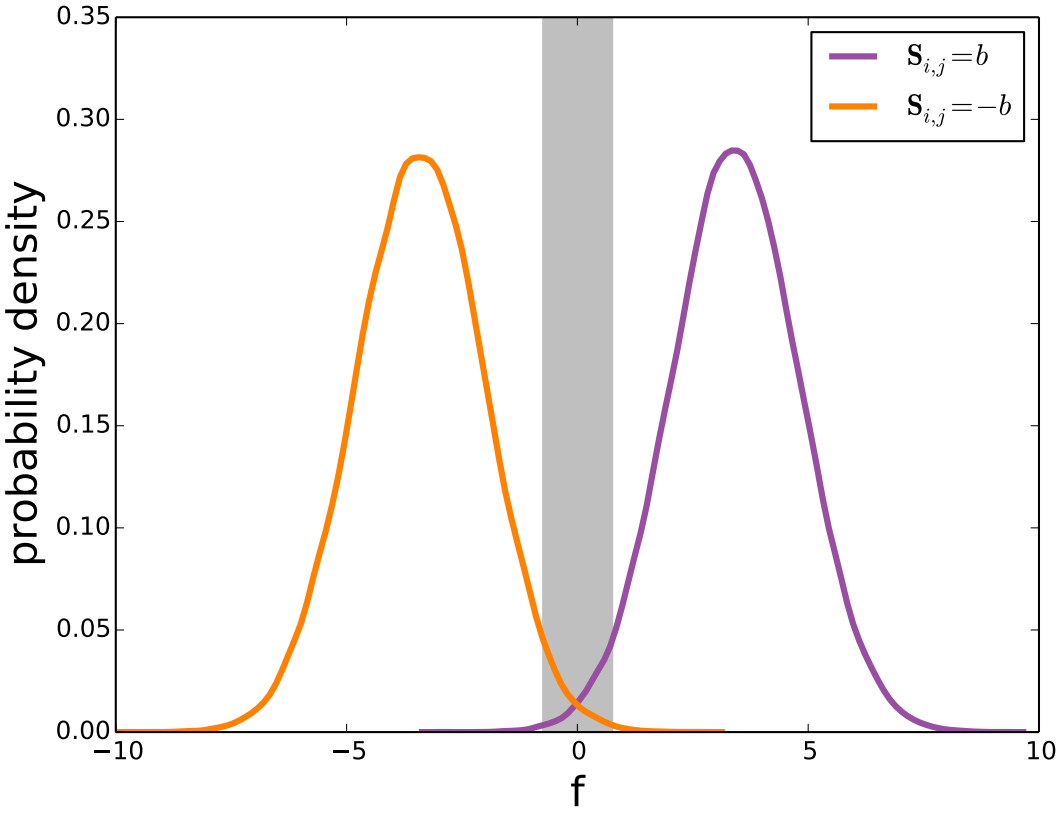

We can determine all large coefficients in one row!

However, it is still hard to learn small coefficients...

Outline

Leaks help a lot!

Attack without leaks

- dim $= n + 1$, short vector of length $\sqrt{b^2 \cdot N_b + N_1 + 1}$
- cost: $> 2^{128}$

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Naive attack with leaks

- dim $= \mathbf{n} + \mathbf{1}$, short vector of length $\sqrt{N_1 + 1}$
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Improved attack with leaks

- dim $= \mathbf{n} - \mathbf{N_b}$, short vector of length $\sqrt{N_1 + 1}$
- cost: 2^{73}

Conclusion

We present a statistical attack against DRS:

- given 100 000 signatures, security is below 80-bits;
- even less with the current progress of lattice algorithms.

Thank you!