Evolution of Cryptanalysis: Security Updates on SHA-3 and AES Hashing

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CRYPTANALYSIS TASKFORCE

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Outlines

1. Introductions to Hash Functions and Cryptanalysis
2. Developments of Cryptanalysis
3. Security Status of SHA-3
4. Security Status of AES Hashing
5. Summary and Projections

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Introduction to Hash Functions

Definition

$h : \{0,1\}^* \longrightarrow \{0,1\}^n$, a function mapping a bit string of arbitrary length to a fixed length n-bit digest.

Security Properties

- **Collision Resistance**: it is computationally difficult to find x, x' such that $h(x) = h(x')$.
- **Preimage Resistance**: given t, it is computationally difficult to find x such that $h(x) = t$.
- **Second-Preimage Resistance**: given x, it is computationally difficult to find x' such that $h(x) = h(x')$.

Applications

digital signatures ($sign_{key}(h(m))$), password storage ($h(salt, password)$), checksum ($h(m)$), Message Authentication Code ($h(key, m)$)

SHA-3

Triggered by the broken of SHA-1 in 2005 by Xiaoyun Wang et al., NIST organized the SHA-3 competition

timeline

- 2008/10: 64 submissions received
- 2009/07: 14 selected for Round 2
- 2010/12: 5 selected for Round 3
- 2012/10: Keccak announced as the winner, designed by Guido Bertoni, Joan Daemen, Michael Peeters, and Gilles Van Assche.
- 2015/08: formally standardized as "SHA-3"

SHA-3 basics

- supports 224, 256, 384, 512-bit digest sizes, and extendable digest sizes by SHAKE-128 and SHAKE-256.
- design strategy different from SHA-1 and SHA-2
- 24 rounds; designers also proposed KangarooTwelve with 12 rounds.

AES Hashing

DES has to be replaced due to linear cryptanalysis by Matsui 1993, and other issues, NIST organized AES competition

timeline

- 1998/06: 15 submissions received
- 1999/08: 5 selected for Round 2
- 2000/10: Rijndael announced as winner, designed by Joan Daemen, Vincent Rijmen
- 2001/11: formally standardized as "AES"

AES basics

- supports 128, 192, 256-bit keys
- wide trail strategy to resist differential cryptanalysis
- 10/12/14 rounds; can be converted to hash function by modes such as Davies-Meyer $H_i = E_M(H_{i-1}) + M$

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Introduction to Cryptanalysis

Cryptanalysis methods develop together with designs, serve as a tool to trade-off between simplicity, efficiency, and security for designs, through round number etc.

Cryptanalysis Methods

differential cryptanalysis, linear cryptanalysis, integral cryptanalysis, meet-in-the-middle attack, boomerang attack, cube attack, etc.

Take differential cryptanalysis as example, the search of high probability differentials are done:

Evolution

- by hand according to cryptanalysts' experiences, e.g., SHA-1 in 2005 ?
- by dedicated search programs
- by SAT like solvers (from Mouha et al. Inscrypt 2011)
- by Machine Learning (from Gohr CRYPTO 2019)

Search of Differentials

The Problem

Given the round number r, find

$$\Delta_{IN} \rightarrow \Delta_1 \rightarrow \Delta_2 \cdots \rightarrow \Delta_r = \Delta_{OUT}$$

such that the overall probability from $\Delta_{IN} \longrightarrow \Delta_{OUT}$ is optimal.

Approaches

- Human experiences used to narrow down the search space: truncated differential search then differential; find good differential gradually by round numbers; meet in the middle approaches, etc.
- SAT-like solvers (solving time hard to predict)
- Machine Learning (constrained by practical complexities)

Optimization by SAT-like solvers

It is hard to achieve optimality for any cryptanalytic attacks, which are usually tunable by various parameters. SAT-like solvers, e.g., SAT, Constrained Programming (CP), Mixed-Integer-Linear-Programming (MILP), work better for man-made search programs.

Simple MILP Example

$$-2x + 2y \geq 1$$
$$-8x + 10y \leq 13$$
$$x, y \geq 0$$
$$x, y \in \mathcal{Z}$$

Objective: maximize $x + y$

Optimal solution is $(x, y) = (1, 2)$ with objective 3.

Differential Search

Constraints: $\Delta_{i-1} \to \Delta_i$ has to be compatible for all i.
Optimization Goal: Overall probability.

Quantum Cryptanalysis

Assuming quantum computer in hand, two major algorithms available

Grover's Algorithm, a.k.a Quantum Search Algorithm

It takes $\Theta(\sqrt{N})$ time by quantum computer to search the one target out of a space of size N, v.s. $\Theta(N)$ by classical computers.
Impacts:

- security strength of block ciphers is halved, 128-bit key becomes insufficient
- (second-) preimage resistance of hash functions is halved

Simon's Algorithm

If the function f is periodic, i.e., $f(x) = f(x + p)$ for all x from the domain, it costs polynomial time to find the period p.

Collision Finding

Collision Attack in Classical Setting

One randomly select x and store $h(x)$ in a hash table until a repeated value detected, this costs $2^{n/2}$ with a hash function with n-bit digest.

Collision Attack in Quantum Setting

One randomly pick $2^{n/3}$ x and store the respective $h(x)$ in quantum memory, and carry out Grover search of $h(y)$ against the stored value, this costs $2^{n/3}$.

- 128-bit digest offers collision resistance of $2^{42.7}$, insufficient ...

Impact on differential-based collision attacks

Lowest useful probability is $2^{-n/2}$ (Classical) v.s. $2^{-2n/3}$ (Quantum); Potentially larger search space, and longer attacked rounds.

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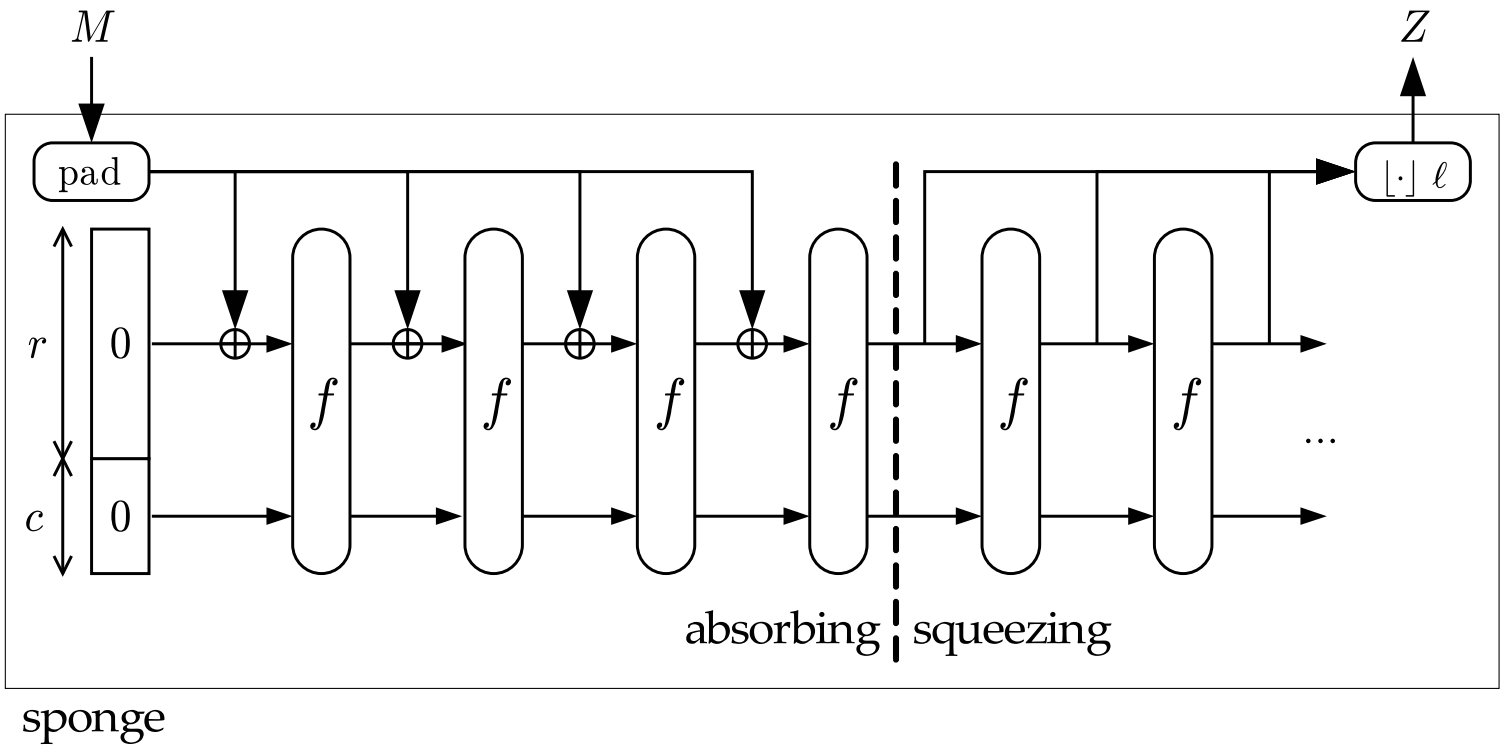
SHA-3 (Keccak) Hash Function

The sponge construction [BDPV11]



- b-bit permutation f
- Two parameters: bitrate r, capacity c, and $b = r + c$.
- The message is padded and then split into r-bit blocks.

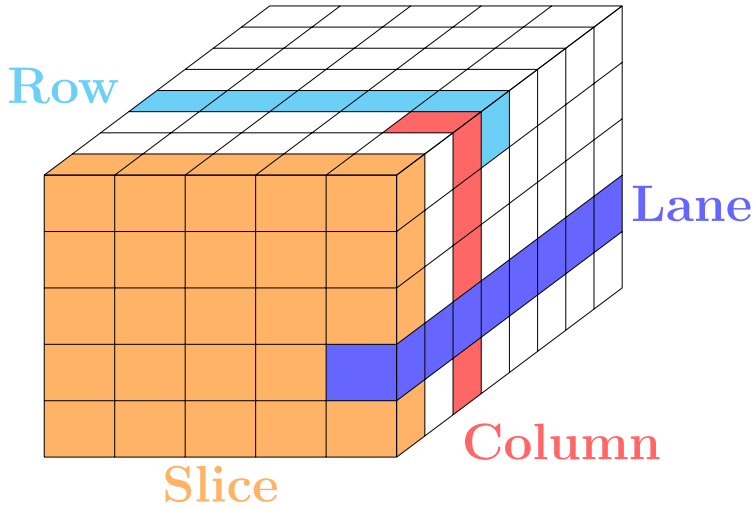
SHA-3 Hash Function

Keccak-f permutation

- 1600 bits: seen as a 5×5 array of 64-bit lanes, $A[x, y], 0 \leq x, y < 5$
- 24 rounds
- each round R consists of five steps:

$$R = \iota \circ \chi \circ \pi \circ \rho \circ \theta$$

- χ : the only nonlinear operation



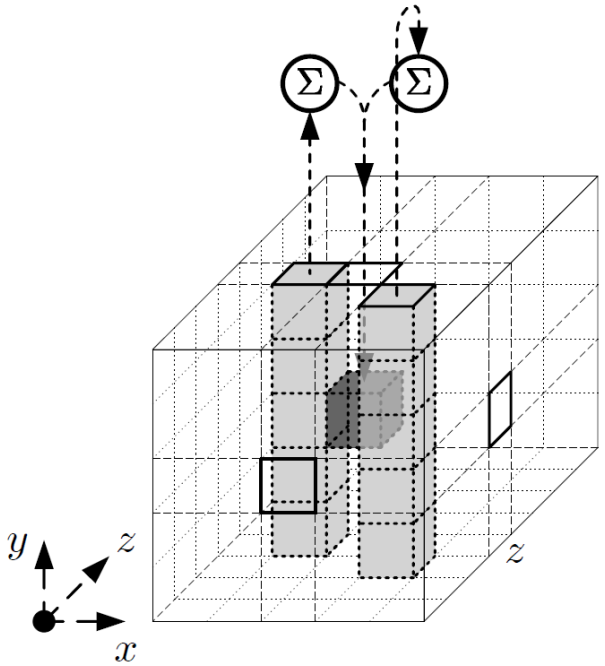
http://www.iacr.org/authors/tikz/

SHA-3 Hash Function

Keccak permutation: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

θ step: adding two columns to the current bit

$$
\begin{aligned}
C[x] =& A[x,0] \oplus A[x,1] \oplus A[x,2] \oplus \\
& A[x,3] \oplus A[x,4] \\
D[x] =& C[x-1] \oplus (C[x+1] \lll 1) \\
A[x,y] =& A[x,y] \oplus D[x]
\end{aligned}
$$



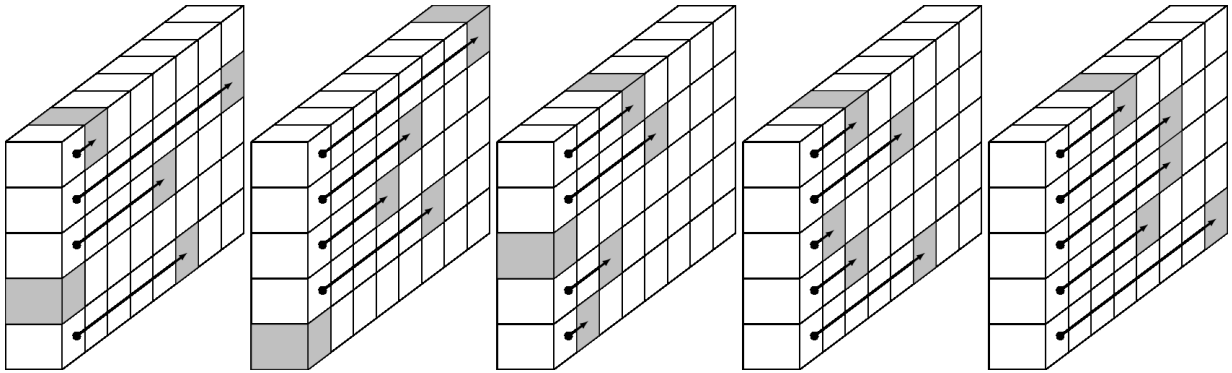
http://keccak.noekeon.org/

- The Column Parity kernel
  - If $C[x] = 0, 0 \leq x < 5$, then the state A is in the CP kernel.

SHA-3 Hash Function

KECCAK permutation: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

ρ step: lane level rotations, $A[x, y] = A[x, y] \lll r[x, y]$



http://keccak.noekeon.org/

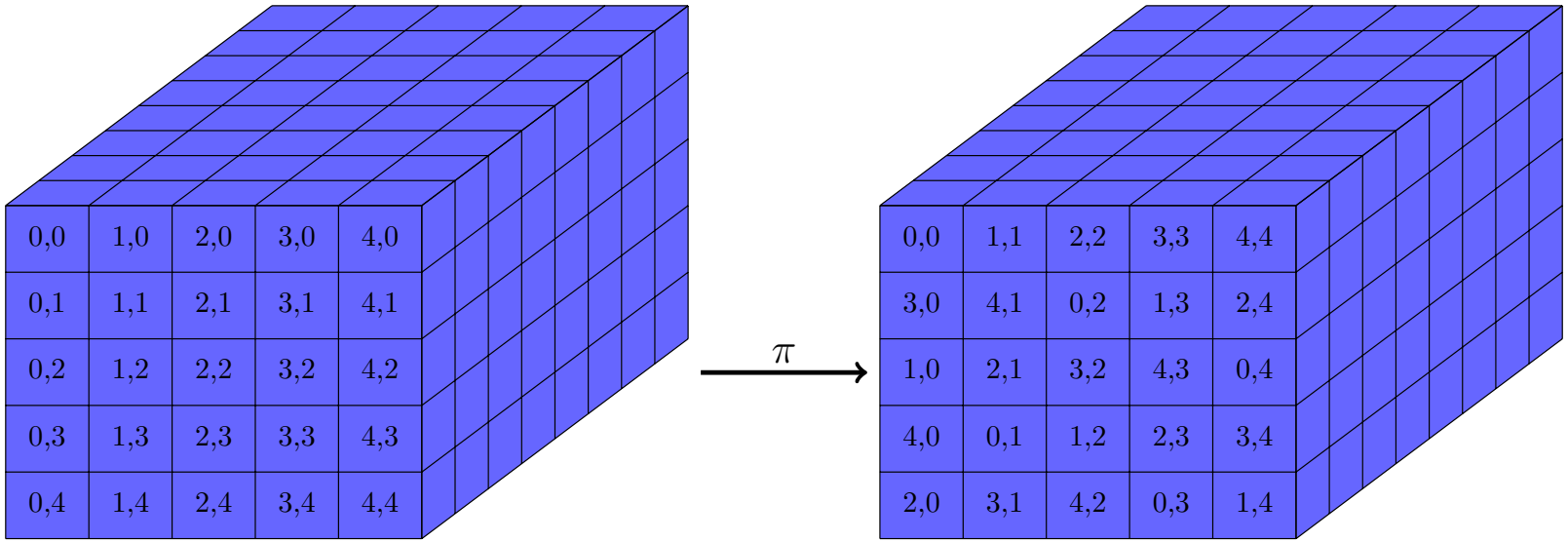
Rotation offsets $r[x, y]$

	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$
$y = 0$	0	1	62	28	27
$y = 1$	36	44	6	55	20
$y = 2$	3	10	43	25	39
$y = 3$	41	45	15	21	8
$y = 4$	18	2	61	56	14

SHA-3 Hash Function

Keccak permutation: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

π step: permutation on lanes



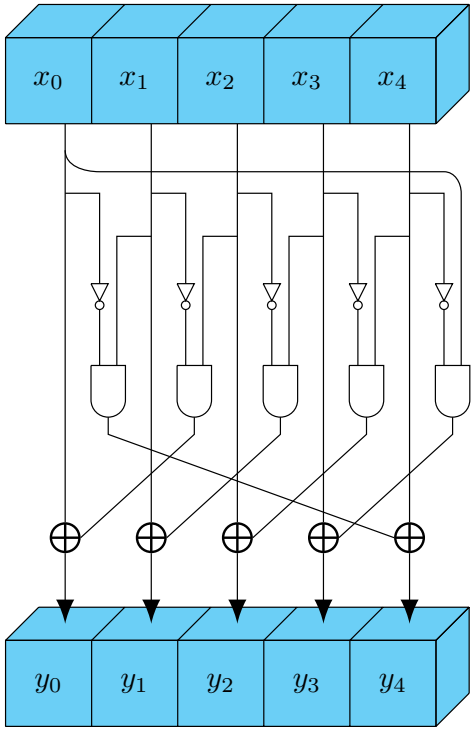
$$A[y, 2 * x + 3 * y] = A[x, y]$$

SHA-3 Hash Function

KECCAK permutation: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

χ step: 5-bit S-boxes, nonlinear operation on rows

$$
\begin{aligned}
y_0 &= x_0 \oplus (x_1 \oplus 1) \cdot x_2 \\
y_1 &= x_1 \oplus (x_2 \oplus 1) \cdot x_3 \\
y_2 &= x_2 \oplus (x_3 \oplus 1) \cdot x_4 \\
y_3 &= x_3 \oplus (x_4 \oplus 1) \cdot x_0 \\
y_4 &= x_4 \oplus (x_0 \oplus 1) \cdot x_1
\end{aligned}
$$



The algebraic degrees of χ and χ^{-1} are 2 and 3.

SHA-3 Hash Function

Keccak permutation: $\iota \circ \chi \circ \pi \circ \rho \circ \theta$

ι step: adding a round constant to the state

Adding one round-dependent constant to the first "lane", to destroy the symmetry.

Without ι

- The round function would be symmetric.
- All rounds would be the same.
- Fixed points exist.
- Vulnerable to rotational attacks, slide attacks, ...

SHA-3 Hash Function

Round function of Keccak-f

Internal state A: a 5×5 array of 64-bit lanes

θ step $C[x] = A[x,0] \oplus A[x,1] \oplus A[x,2] \oplus A[x,3] \oplus A[x,4]$
$D[x] = C[x-1] \oplus (C[x+1] \lll 1)$
$A[x,y] = A[x,y] \oplus D[x]$

ρ step $A[x,y] = A[x,y] \lll r[x,y]$
- The constants $r[x,y]$ are the rotation offsets.

π step $A[y, 2*x+3*y] = A[x,y]$

χ step $A[x,y] = A[x,y] \oplus ((\ A[x+1,y]) \& A[x+2,y])$

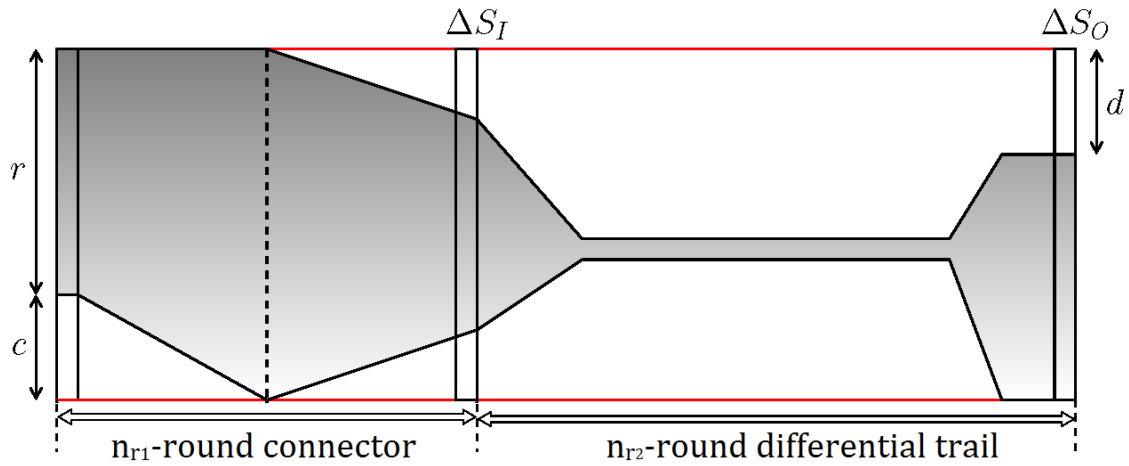
ι step $A[0,0] = A[0,0] \oplus RC$
- $RC[i]$ are the round constants.

$L \triangleq \pi \circ \rho \circ \theta$

The only non-linear operation is χ step.

Collision Attacks — the Framework

$(n_{r_1} + n_{r_2})$-round collision attacks:



- n_{r_1}-round **connector**: produces message pairs (M_1, M_2) s.t.

$$\mathrm{R}^{n_{r_1}}(\overline{M_1}||0^c) + \mathrm{R}^{n_{r_1}}(\overline{M_2}||0^c) = \Delta S_I, \quad (\mathrm{R}^{n_{r_1}} : n_{r_1} \text{ rounds})$$

$$n_{r_1} = 1 \text{ [DDS13]} \longrightarrow n_{r_1} = 2 \text{ [QSLG17]} \longrightarrow n_{r_1} = 3 \text{ [SLG17]} .$$

- n_{r_2}-round **differential**: $\Delta S_I \to \Delta S_O$,
  with first d bits of ΔS_O being 0, *i.e.*, collision.

Collisions Attack — The State of the Art

Without the help of SAT or MILP

Rounds	Target	Setting	Complexity	Reference
5	SHA3-256	Classical	Practical	[GLL+19]
5	SHA3-224	Classical	Practical	[SLG17]
5	SHAKE128	Classical	Practical	[QSLG17]
4	SHA3-384	Classical	2^{147}	[DDS13]
3	SHA3-512	Classical	Practical	[DDS13]

Collisions Attack — The State of the Art

With the help of SAT or MILP

Rounds	Target	Setting	Complexity	Reference
6	SHA3-256	Quantum	$2^{104}/\sqrt{S}$	[GLST22]
6	SHA3-224	Quantum	$2^{98}/\sqrt{S}$	[GLST22]
6	SHAKE128	Quantum	$2^{67}/\sqrt{S}$	[GLST22]
6	SHAKE128	Classical	$2^{123.5}$	[GLST22]
5	SHA3-256	Classical	Practical	[GLL+19]
5	SHA3-224	Classical	Practical	[SLG17]
5	SHAKE128	Classical	Practical	[QSLG17]
4	SHA3-384	Classical	2^{147}	[DDS13]
3	SHA3-512	Classical	Practical	[DDS13]

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Description of AES

There are 10/12/14 rounds for AES-128/192/256, respectively.

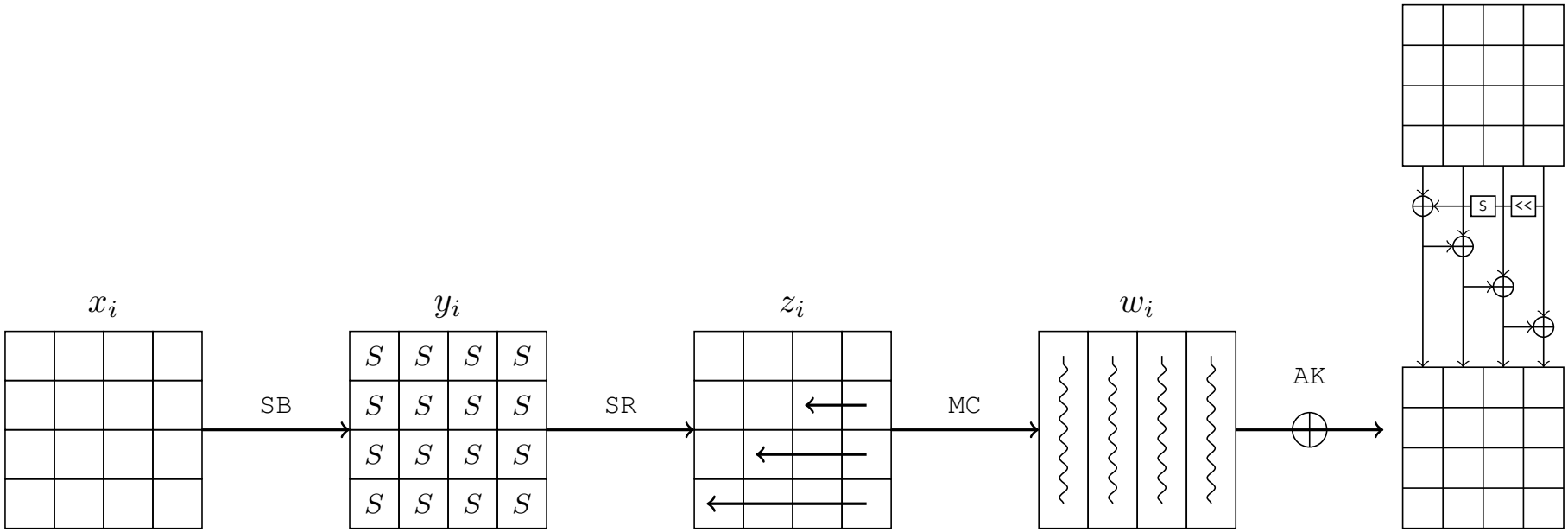

Figure: The round function of AES

(Triangulating) Rebound Attack

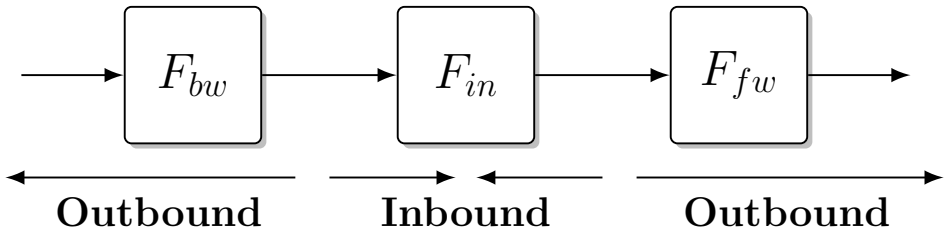

Figure: The Rebound Attack

- introduced by Mendel et al. in FSE 2009
- relies on high probability differentials in F_{bw} and F_{fw}
- constraints: the middle have to match in state and keys (if use)
- optimization goal: overall complexity

Collision Attack on AES Hashing

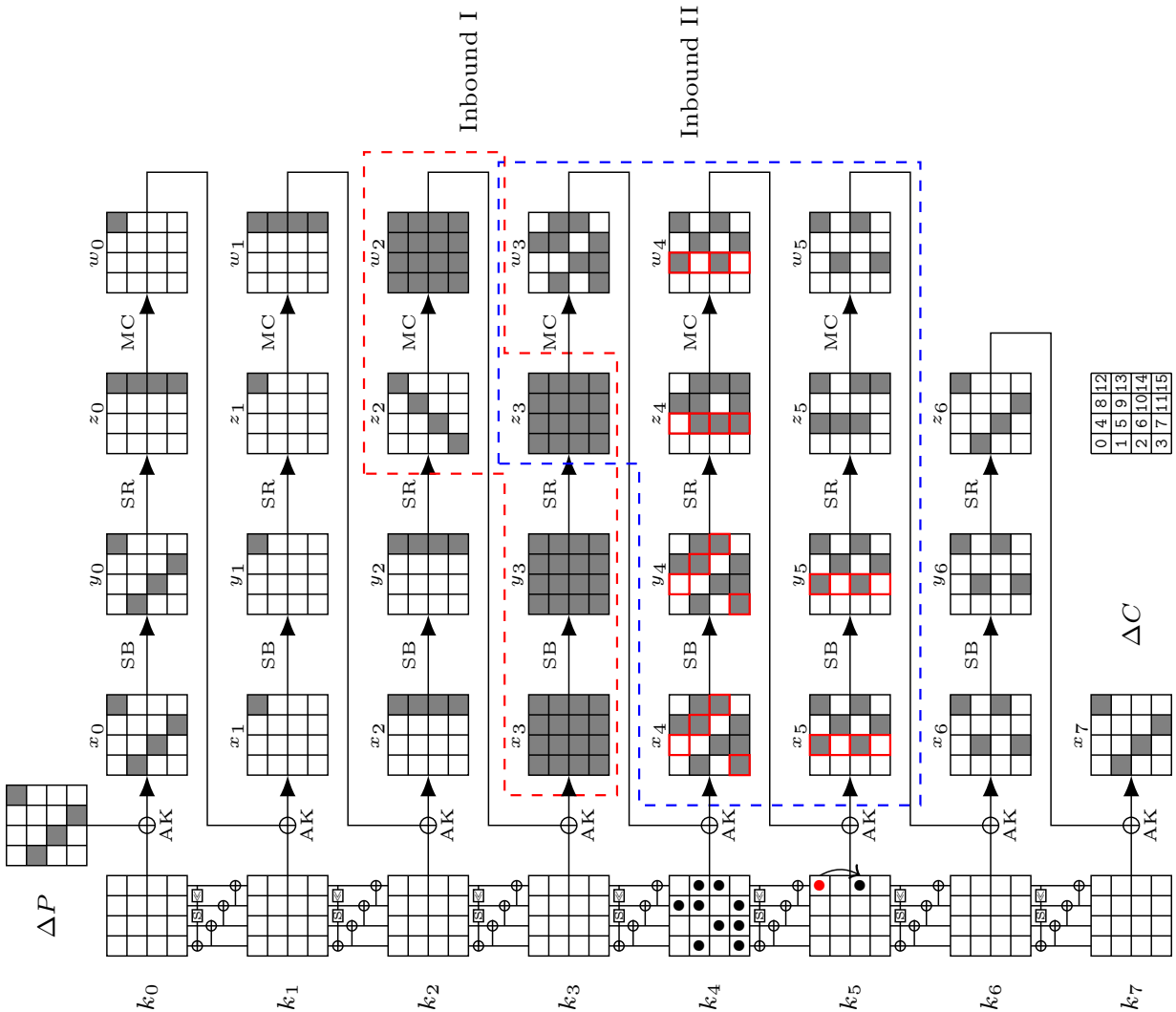

Figure: Semi-free-start collision attack on 7-round AES-128 with complexity 2^{56} in classical setting.

Collision Attack on AES Hashing

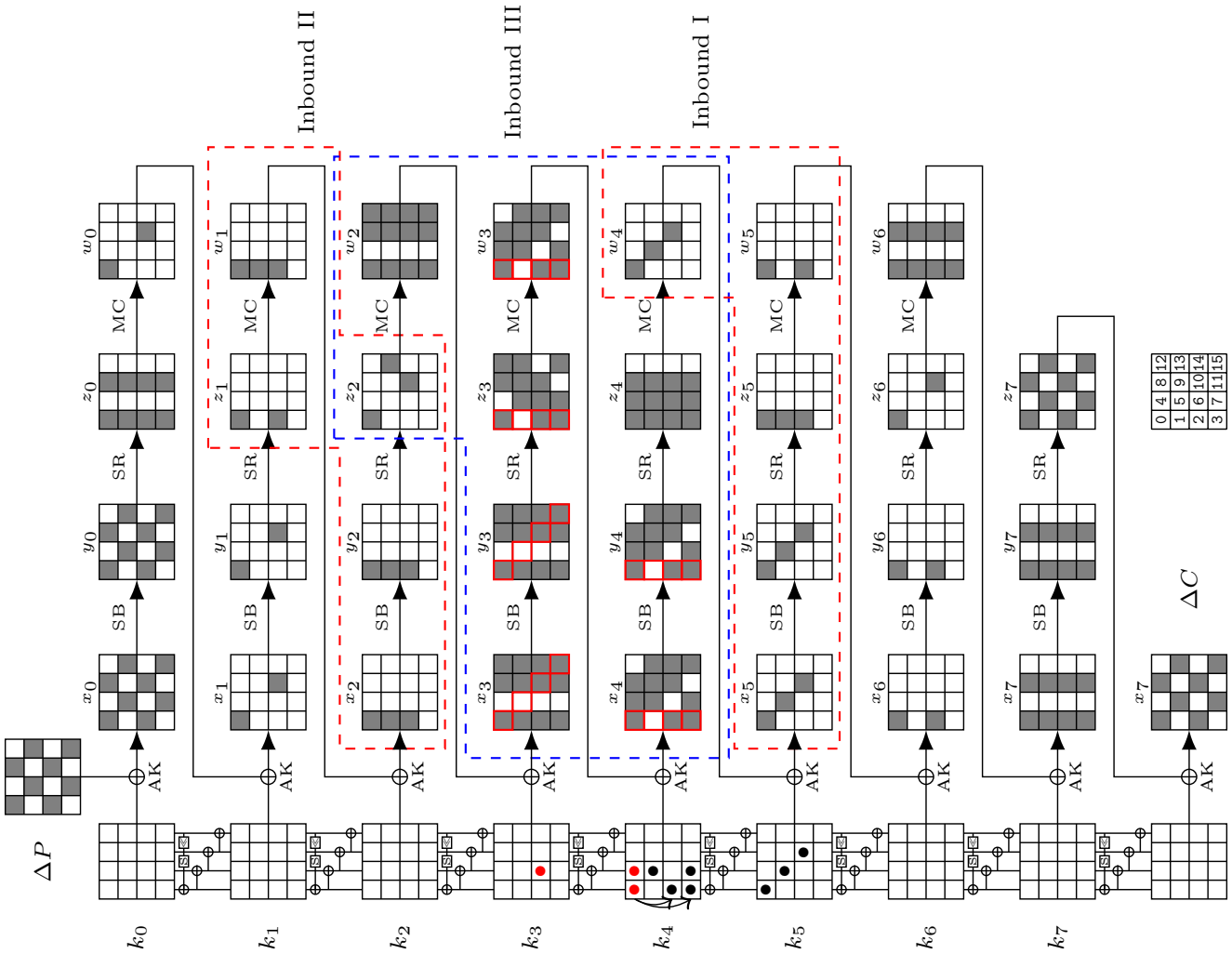

Figure: Semi-free-start collision attack on 8-round AES-128 with complexity 2^{34} in quantum setting.

Collision Attack on AES Hashing — Summary

- Classical: 7-round with complexity 2^{56}
- Quantum: 8-round with complexity 2^{34}

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Cryptanalysis by Machine Learning

The idea is to mimic the differential based key recovery attack: to train the machine to distinguish the reduced rounds with high-probability differential from random ones, and then launch key recovery attack utlizing such distinguishers.

Developments

- First Attempt by Gohr at CRYPTO 2019 with application to SPECK
- Second Attempt by Bao et al. ASIACRYPT 2022 to SPECK and SIMON with better complexities than men-made attacks.

Limitations

Limited by practical complexity, no good theoretical model to treat the complexities, hard to project.

Conclusions

- Collision attack of SHA-3 is up to 6 rounds, lots of security margin left, maybe KangarooTwelve is a good idea (twice as fast).

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- Collision attack of SHA-3 is up to 6 rounds, lots of security margin left, maybe KangarooTwelve is a good idea (twice as fast).
- Collision attack of AES is up to 7/8 rounds in classical and quantum settings, still usable, but too low security strength in quantum setting.

Conclusions

- Collision attack of SHA-3 is up to 6 rounds, lots of security margin left, maybe KangarooTwelve is a good idea (twice as fast).
- Collision attack of AES is up to 7/8 rounds in classical and quantum settings, still usable, but too low security strength in quantum setting.
- What to expect from future development of cryptanalysis: deeper look at the core algorithms underlying the solvers and machine learning

Thank You !

Q&A

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